Intermediate Microeconomics

Chapter 6

Choice under Uncertainty

Uncertainty

- Until now, the situation in which the consumer would find herself was certain
- *State of the world* = outcome of an uncertain situation
- *Contingent commodity* = commodity whose level depends on which state of the world occurs (e.g., consumption if a bet is won, or if a bet is lost)

Expected value

- Each state of the world can occur with a certain chance called *probability*
- Suppose we consider a bet
- *Expected value* = outcome that we would obtain, on the average, from playing the bet an infinity of times
- Algebraically, if p_i is the probability of state of the world i and w_i is the outcome in that state:

$$E(w) = p_1 w_1 + p_2 w_2 + \dots + p_n w_n$$

Choice under uncertainty

- St. Petersburg paradox (Bernoulli, 1738)
  - a coin is tossed until "head" appears (toss n)
  - payoff from participating: $R(n) = 2^n$
  - how much would you pay as entry fee?
- What is different when there is uncertainty? Risk matters!

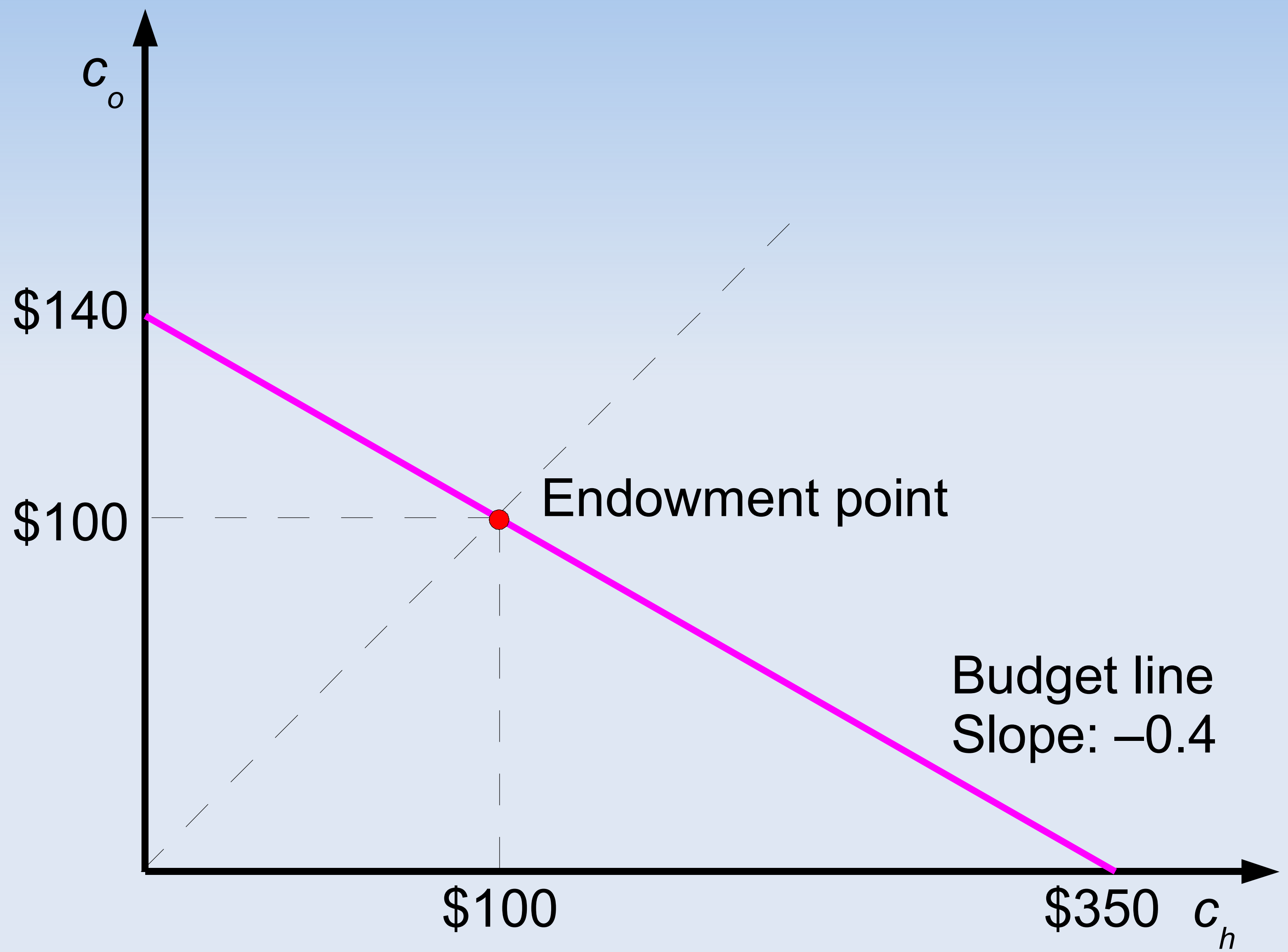
Contingent commodities

- Suppose we consider a bet: next card drawn from the deck is not heart
  - if true, earn \$0.40 on the dollar
  - if false, lose \$1 on the dollar
- Initial endowment is \$100
  - consumption if next card is heart = c_h
  - consumption if next card is different suit = c_o
- If no bet is placed: $c_h = c_o$ = \$100 (endowment point, on the 45 degree line)

An example of uncertainty

- In principle, you could bet both on ”next card is not a heart” and against it
- If the payoff structure is the same, regardless of the bet, then the budget line is a straight line
- The slope of the budget line is given by the negative of the ratio of the potential wins/losses: $-0.4/1 = -0.4$
- The endowment point (no bet) is always an alternative $\Rightarrow$ has to lie on the budget line

Contingent commodities



Fair gambles

- *Actuarially fair gamble* = gamble for which the expected monetary gain is zero
- *Odds* = the ratio of the probability of two events occurring
- *Fair odds line* = budget constraint that reflects the opportunities presented by an actuarially fair gamble (i.e., the ratio of the probabilities is equal to the inverse of the ratio of the corresponding gains/losses)

Back to our example

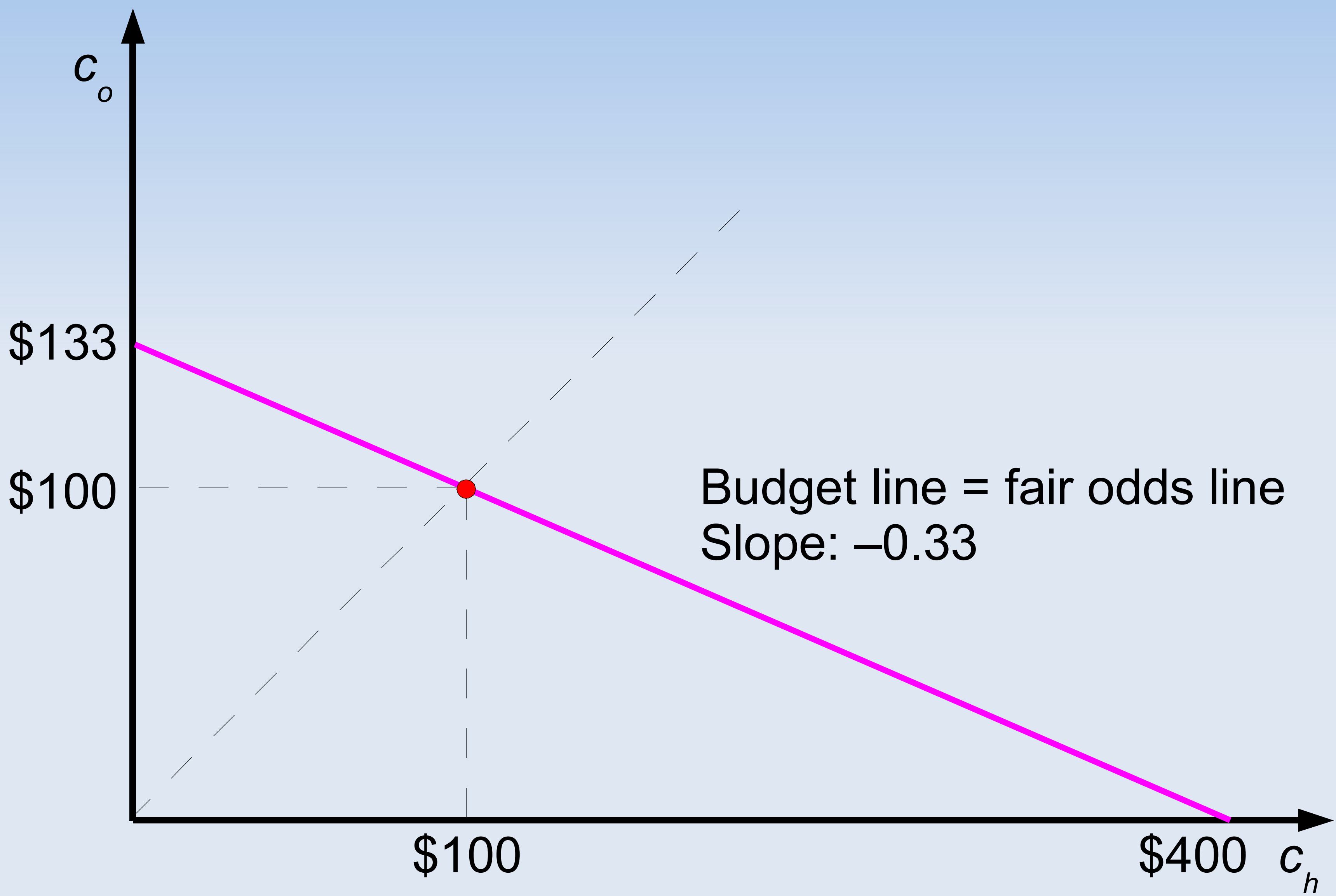
- Is our example a fair bet?
  - a heart is drawn: chance = ¼, loss = $1
  - other suit drawn: chance = ¾, gain = $0.4
  - on the average: $\frac{3}{4} \times 0.4 - \frac{1}{4} \times 1 = \$0.05$

  $\Rightarrow$ you gain $0.05 on each bet
- What would be a fair bet? Suppose you gain *d* dollars per dollar when another suit shows:

$$\tfrac{3}{4} \times d - \tfrac{1}{4} \times 1 = 0 \;\Rightarrow d = \tfrac{1}{4} \div \tfrac{3}{4} = 0.33$$

- The budget line would now have a slope of 0.33 too (¼ ÷ ¾) – the fair odds line

Fair odds line



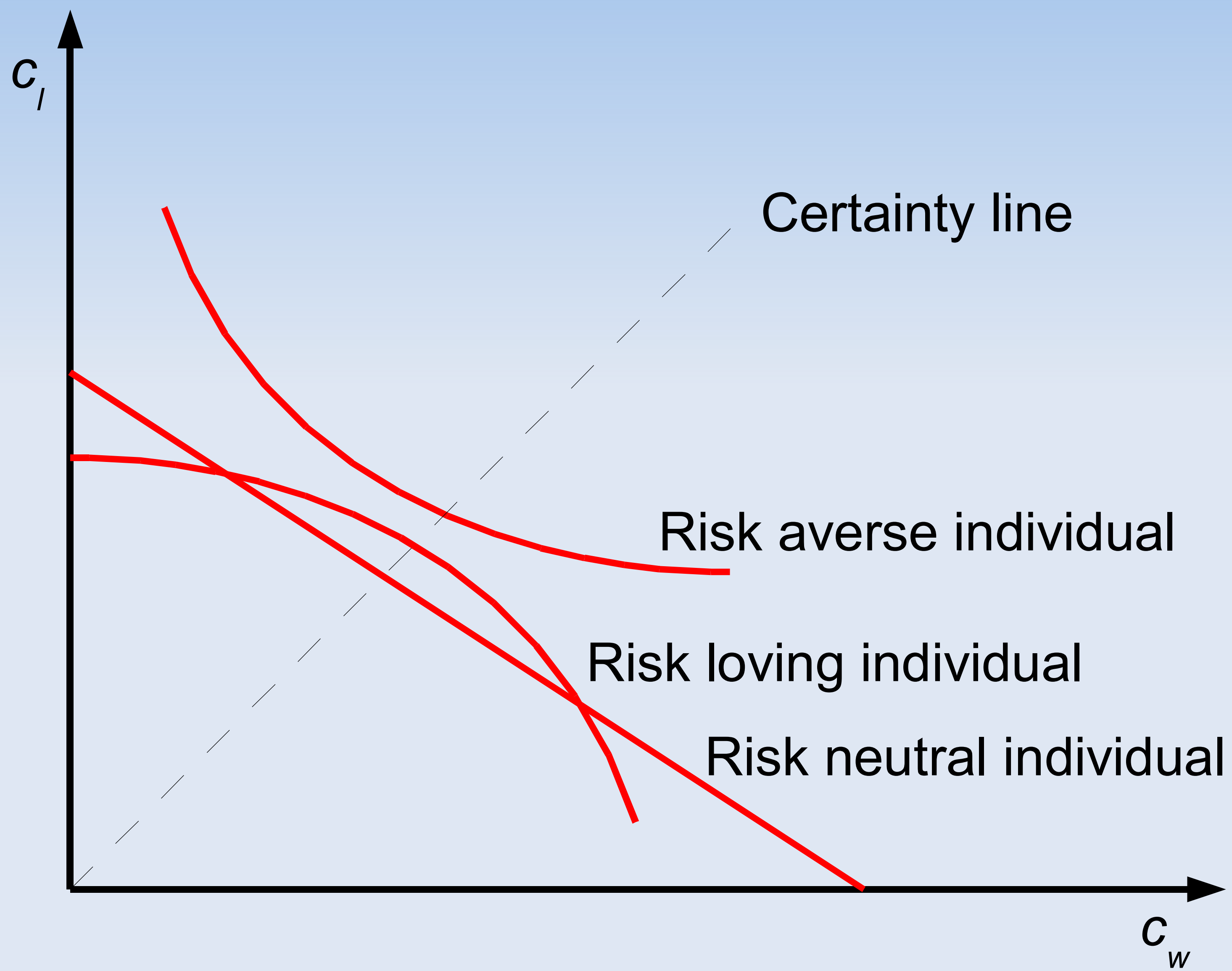
Risk aversion

- *Risk averse* = individual who would not accept an actuarially fair gamble (risk is bad)
- *Risk lover* = individual who prefers a gamble with a certain expected value to certainty with the same expected value (risk is good)
- *Risk neutral* = individual who is indifferent among alternatives with the same expected value (risk does not matter)
- Most individuals are risk averse

Preferences for risk

- *Risk averse* = indifference curves look as before
- *Risk lover* = indifference curves bow outward
- *Risk neutral* = indifference curves are straight lines, parallel to the fair odds line
- *Certainty line* = 45 degree line
- Most individuals are risk averse $\Rightarrow$ will focus only on risk averse individuals

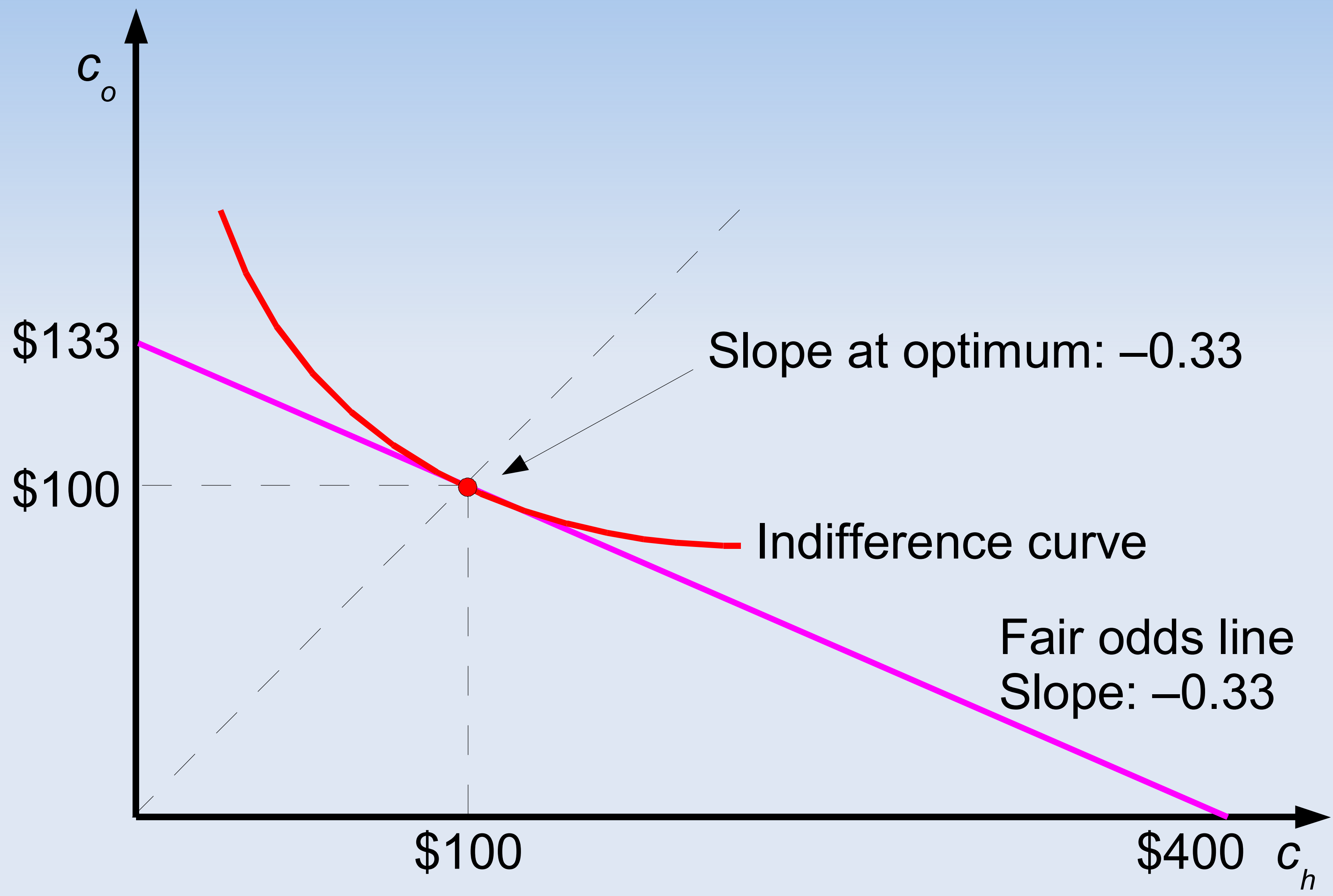
Indifference curves



Equilibrium

- Risk averse individuals prefer certainty to risk
- Hence, when presented to a risky alternative that has the same expected value as a certain alternative, they will *always* choose the certain alternative
- This means that they will always choose the endowment point when faced with a fair odds line $\Rightarrow$ the slope of their indifference curves on the 45 degree line is equal to the odds ratio

Equilibrium for risk averse people



Equilibrium

- When faced with a non-actuarially fair bet, risk averse individuals might choose to place *some* money on the bet
- In contrast, risk lovers will choose to place *all* their money on the bet
- Las Vegas and Atlantic City suggest that there are many risk-loving individuals
- But: most of them place small bets – so maybe not risk lovers after all...

Applications

- *Risk premium* = extra return on an investment, to compensate for risk

$$RP = E(I) - r_f$$

- *Diversification* = process of buying several assets in order to reduce risk ("don't put all your eggs in one basket")
- More on these topics: ECON435 (Financial Markets)

Insurance

- In many cases, people cannot choose whether to take a risk or not $\Rightarrow$ take insurance
- *Premium* = price of obtaining ($1 worth of) insurance coverage
- *Actuarially fair insurance* = premium equals the expected payout of the insurer

Example of insurance problem

- Suppose you have a car and there is a probability p of getting into an accident
- If no accident occurs, then have \$$a$; if an accident occurs, need to spend \$$a$ on repairs
- Consumption when there's no accident is c_g (”good” state) and when there's an accident is c_b (”bad” state)
- The ”endowment point” is then the intercept with the ”good state” axis

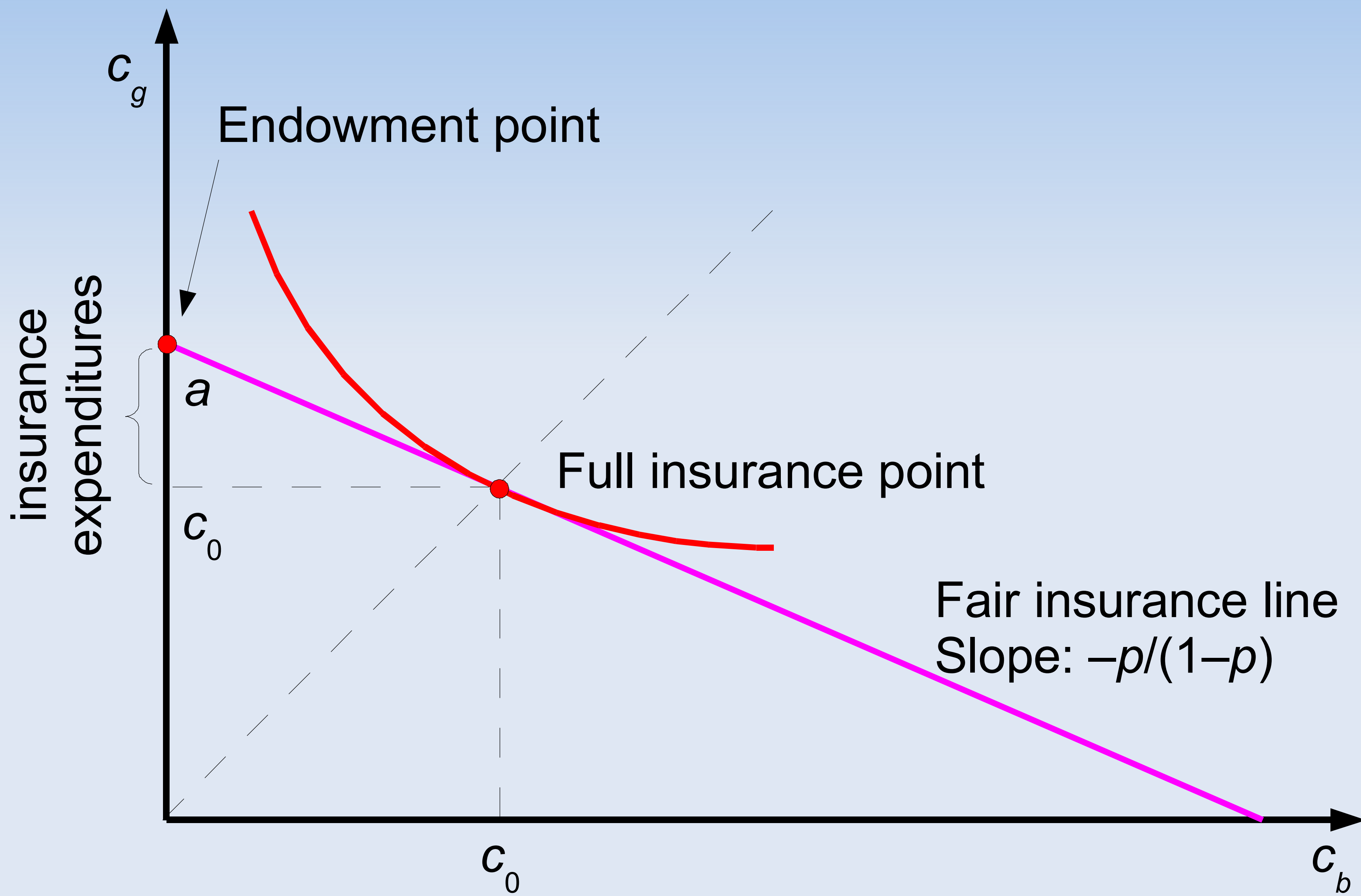
Fair insurance

- An actuarially fair insurance pays as much as its premium, on average $\Rightarrow$ for \$1 of coverage:

$$(1 - r)\,p = r\,(1 - p) \Rightarrow r = p$$

- A risk-averse person would always choose full insurance ($c_g = c_b = c_0$, on the 45 degree line) when presented with an actuarially fair insurance
- Hence, insurance expenditures = a – c_0

Equilibrium with fair insurance



Coverage with unfair insurance

- What happens if $r > p$? Insurance is unfair (you pay more, on average, than get repaid by the insurance company)
- But is this really ”unfair”? Insurance companies need to recover their costs
- Risk averse individuals still buy some insurance (but not full)
- The budget line is now steeper than the fair odds line (because the negative of the slope is $r / (1 - r) > p / (1 - p)$)

Equilibrium with unfair insurance