

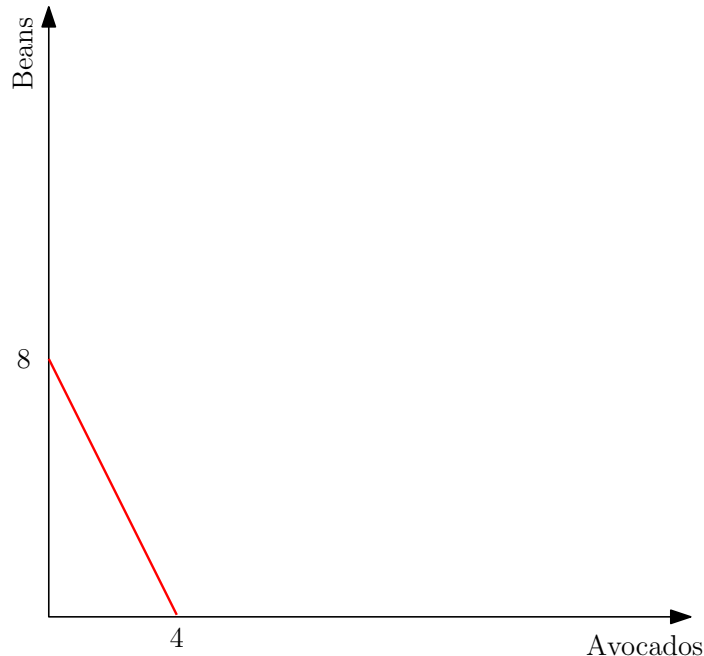
Econ306–Intermediate Microeconomics
Fall 2007
Midterm exam — Solutions

Question 1 (*12 points*)

Perry lives on avocado and beans. The price of avocados is \$10, the price of beans is \$5, and his income is \$40.

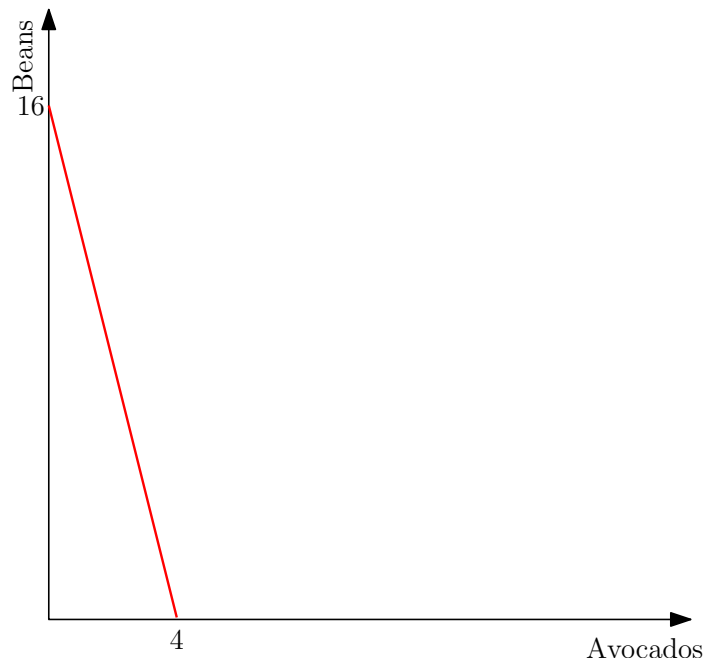
(i) (*3 points*) Show Perry's budget line on a graph with avocados on the horizontal axis and beans on the vertical axis. Calculate the intercepts with the two axes and the slope of the budget line.

Answer: If Perry spent all his income on avocados, he could buy $\frac{40}{10} = 4$ avocados; if he spent all his income on beans, he could buy $\frac{40}{5} = 8$ beans. These are the two intercepts of the budget line. The slope of the budget line is given by the negative of the price ratio: $-\frac{10}{5} = -2$. The budget line will look like in the graph below.



(ii) (3 points) Draw another budget line showing what Perry's budget would be if his income doubled, the price of avocados doubled, and the price of beans stayed the same. Calculate the intercepts with the two axes and the slope of this new budget line.

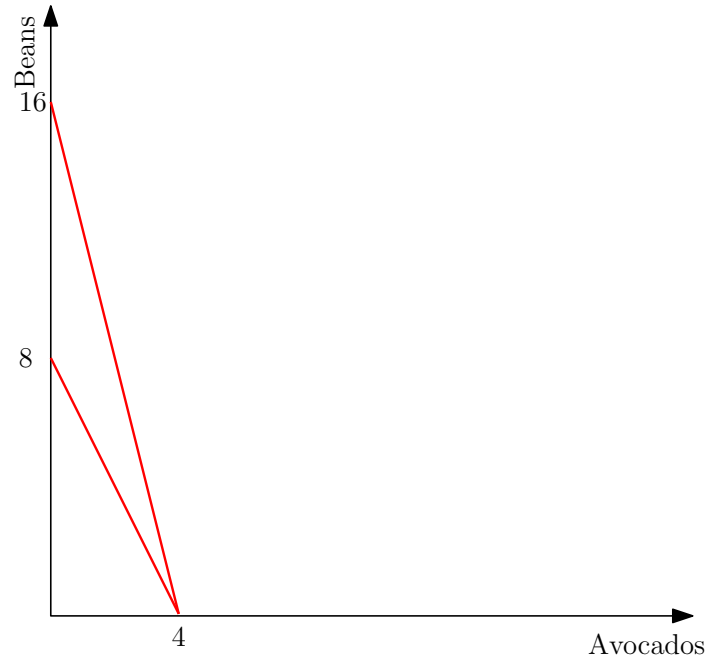
Answer: In this case, Perry's income becomes $40 \cdot 2 = \$80$ and the price of avocados is \$20. If Perry spent all his new income on avocados, he could buy $\frac{80}{20} = 4$ avocados, again; if he spent all his income on beans, he could now buy $\frac{80}{5} = 16$ beans. The slope of the new budget line is $-\frac{20}{5} = -4$. The new budget line will look like in the graph below.



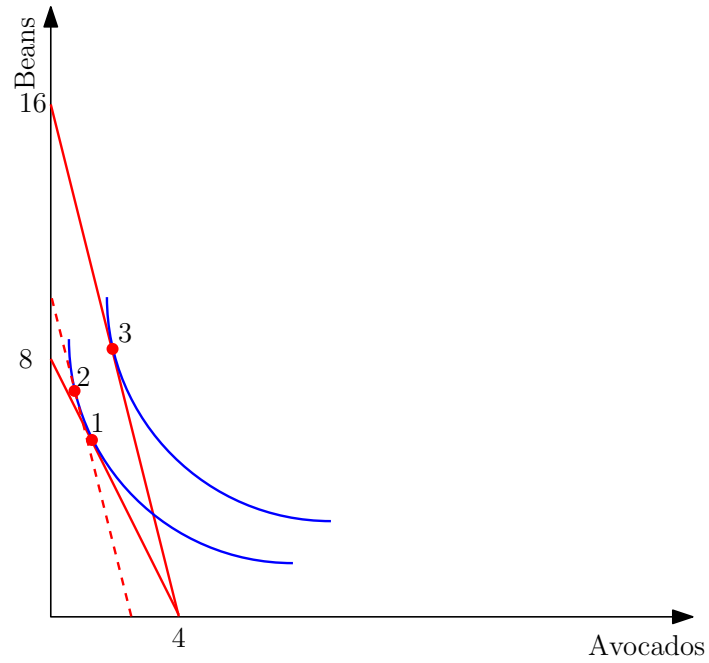
(iii) (6 points) If Perry's income stayed the same at \$40, which price or prices should change and by how much in order for him to face the same budget constraint as in the previous question? If the good(s) whose price changed are normal, show on the graph the substitution and the income effects and explain how the quantity consumed of the good(s) changed.

Answer: In order to better answer this question, draw the two budget lines together on the same graph. It becomes clear now that the second budget line could have been the result of a fall in the price of beans (to be more precise, for Perry to afford a maximum quantity of 16 beans when his income is still \$40, the price of beans must have fallen to $\frac{40}{16} = \$2.5$).

In this case, the substitution effect and income effect are measured on the *vertical* axis, the axis that represents beans. The substitution effect (shown in the second graph as the move from point 1 to point 2) pushes Perry to consume more beans, since they became



cheaper. As Perry can afford the same quantity of beans as before, but while spending less, he feels as if he has more money and as such can afford to buy more beans (because beans are a normal good). This is the income effect, shown in the graph as the move from point 2 to point 3.



Question 2 (10 points)

Max has the utility function $U(x, y) = x(y + 1)$. The price of x is \$2 and the price of y is \$1. Income is \$10.

(i) (3 points) Derive the marginal utility of both goods.

Answer: The marginal utility of a good is the derivative of the utility function with respect to the quantity of that good consumed:

$$\begin{aligned} MU_x &= \frac{\partial U(x, y)}{\partial x} = y + 1, \\ MU_y &= \frac{\partial U(x, y)}{\partial y} = x. \end{aligned}$$

(ii) (4 points) Find the optimal consumption bundle: how much x and how much y does Max demand? What is the marginal rate of substitution at the optimum?

Answer: At the optimal consumption bundle, the ratio of the marginal utilities is equal to the ratio of the corresponding prices:

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y} \Rightarrow \frac{y + 1}{x} = \frac{2}{1} \Rightarrow y + 1 = 2x \Rightarrow y = 2x - 1.$$

We also know that Max spends all his income on consumption, so the budget constraint must hold:

$$p_x \cdot x + p_y \cdot y = I \Rightarrow 2x + y = 10.$$

Substituting for y in the equation above yields:

$$2x + (2x - 1) = 10 \Rightarrow 4x - 1 = 10 \Rightarrow 4x = 11 \Rightarrow x = \frac{11}{4} = 2.75.$$

Now we can calculate the optimal quantity of good y :

$$y = 2x - 1 = 2 \cdot 2.75 - 1 = 4.5.$$

Hence, the optimal consumption bundle consists of 2.75 units of good x and 4.5 units of good y .

(iii) (3 points) If his income doubles and prices stay unchanged, will Max's demand for both goods double?

Answer: When income doubles, the only change in the analysis above is in the budget constraint:

$$p_x \cdot x + p_y \cdot y = I \Rightarrow 2x + y = 20.$$

We can again substitute for y in the budget constraint:

$$2x + (2x - 1) = 20 \quad \Rightarrow \quad 4x - 1 = 20 \quad \Rightarrow \quad 4x = 21 \quad \Rightarrow \quad x = \frac{21}{4} = 5.25.$$

Now we can again calculate the optimal quantity of good y :

$$y = 2x - 1 = 2 \cdot 5.25 - 1 = 9.5.$$

Note that the doubling of the income lead to a less than double optimal quantity of good x , but to a more than double optimal quantity of good y .

Question 3 (10 + extra 5 points)

Patience has a utility function depending on her consumption today c_0 and on her consumption tomorrow (next period) c_1 . She will earn 100 units of consumption in each period (i.e., $I_0 = I_1 = 100$). She can borrow or lend at an interest rate of 10%.

(i) (1 point) Write down her intertemporal budget constraint.

Answer: The intertemporal budget constraint is

$$c_0 + \frac{c_1}{1+i} = I_0 + \frac{I_1}{1+i} \quad \Rightarrow \quad c_0 + \frac{c_1}{1.1} = 100 + \frac{100}{1.1} \quad \Rightarrow \quad c_0 + \frac{c_1}{1.1} = 190.91.$$

(ii) (3 points) Calculate the intercepts of the budget line with the horizontal (consumption today) and with the vertical (consumption tomorrow) axes, as well as the slope of the budget line.

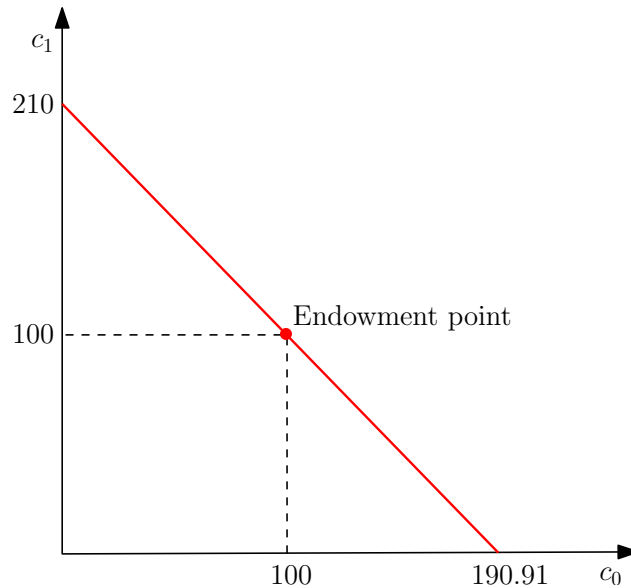
Answer: If Patience spent all her income on consumption today ($c_1 = 0$), she would be able to afford 190.91 units of consumption. If she spent all her income on consumption tomorrow ($c_0 = 0$), her consumption would be given by

$$\frac{c_1}{1.1} = 190.91 \quad \Rightarrow \quad c_1 = 190.91 \cdot 1.1 = 210.$$

The slope of the intertemporal budget constraint is given by the negative of $(1+i)$, and so it is -1.1 .

(iii) (1 point) Graph the budget line, making sure to emphasize her endowment point.

Answer: The endowment point represents her affordable consumption if she neither saves nor borrows. In that case, $c_0 = I_0 = 100$ and $c_1 = I_1 = 100$.



(iv) (5 points) Suppose that Patience chose to consume 41 units of consumption today. How would she be affected by a drop in the interest rate from 10% to 7.5%? Make sure to describe the substitution and the income effects.

Answer: A drop in the interest rate is equivalent to an increase in the price of future consumption $\left(\frac{1}{1+i}\right)$. In this case, the substitution effect implies that she would consume less in the future, because it is more expensive, and more today. This in turns pushes Patience to save less.

In order to determine the income effect, we need to figure out if Patience is a saver or a borrower. Since she consumes today less than her income ($41 < 100$), she is a saver. In this case, the lower interest rate means that she will receive less in the future as interest payments and she is negatively affected (has less money). As a result, she will lower her consumption both today and tomorrow and save more.

The final effect of the drop in the interest rate depends on which of the two effects dominates. If the substitution effect is stronger, then she will end up saving less. If the income effect is stronger, then she will save more.

(v) (extra 5 points) Suppose Patience's utility function is of the form $U(c_0, c_1) = \sqrt{c_0} + 2\sqrt{c_1} = c_0^{1/2} + 2c_1^{1/2}$. What is the marginal utility of consumption today and of consumption tomorrow? Calculate the optimal allocation of consumption between the two periods (i.e., c_0 and c_1).

Answer: The two marginal utilities are calculated as the derivative of the utility function with respect to the two types of consumption, today and in the future:

$$MU_0 = \frac{\partial U(c_0, c_1)}{\partial c_0} = \frac{1}{2\sqrt{c_0}},$$

$$MU_1 = \frac{\partial U(c_0, c_1)}{\partial c_1} = \frac{2}{2\sqrt{c_1}} = \frac{1}{\sqrt{c_1}}.$$

Next, remember the ratio of the marginal utilities is equal to the ratio of the prices at the optimal consumption point. The price of c_0 is equal to 1 and the price of c_1 is $\frac{1}{1+i}$. Then,

$$\frac{MU_0}{MU_1} = \frac{1}{\frac{1}{1+i}} \Rightarrow \frac{\frac{1}{2\sqrt{c_0}}}{\frac{1}{\sqrt{c_1}}} = \frac{1}{1+i} \Rightarrow \frac{\sqrt{c_1}}{2\sqrt{c_0}} = 1+i \Rightarrow \frac{\sqrt{c_1}}{2\sqrt{c_0}} = 1.1 \Rightarrow$$

$$\sqrt{c_1} = 2\sqrt{c_0} \cdot 1.1 \Rightarrow \sqrt{c_1} = 2.2\sqrt{c_0} \Rightarrow c_1 = 2.2^2 \cdot c_0 \Rightarrow c_1 = 4.84c_0.$$

Now we can substitute for c_1 in the budget constraint to find the value of c_0 :

$$c_0 + \frac{c_1}{1.1} = 190.91 \Rightarrow c_0 + \frac{4.84c_0}{1.1} = 190.91 \Rightarrow c_0 + 4.4c_0 = 190.91 \Rightarrow$$

$$5.4c_0 = 190.91 \Rightarrow c_0 = \frac{190.91}{5.4} = 35.35.$$

Finally, we can now calculate the optimal consumption in the future:

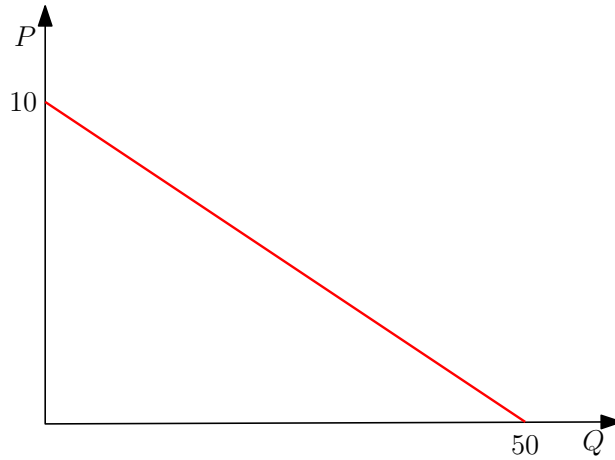
$$c_1 = 4.84c_0 = 4.84 \cdot 35.35 = 171.11.$$

Question 4 (10 points)

Beth works for the federal government and her job is to analyze the market for oranges.

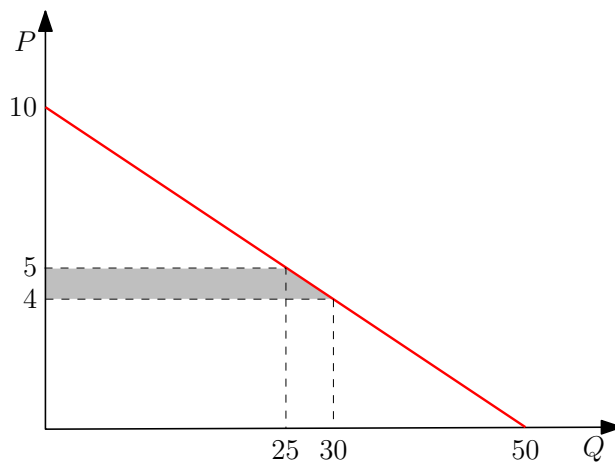
(i) (2 points) Suppose she found that the demand function is $Q = 50 - 5p$, where Q is the quantity of oranges demanded and p is the ongoing price per pound. Graph the demand curve (remember to label the intercepts).

Answer: When the price of oranges is zero, total demand is $Q = 50$. When total demand is zero, the price is $\frac{50}{5} = 10$. The demand curve then looks like in the graph below.



(ii) (4 points) The federal government is considering imposing some restrictions on the production of oranges because of environmental concerns. The current price is \$4 per pound, but after the restrictions it would increase to \$5 per pound. What is the consumer surplus before and after the restrictions? What is the loss in consumer surplus due to the restrictions? If a consumer advocacy group would try to bribe Beth to convince federal authorities that the restrictions should not be imposed, how much would they be willing to spend?

Answer: When the price is $p_0 = \$4$ per pound, the quantity demanded is $Q_0 = 50 - 4 \cdot 5 = 30$ and the consumer surplus equals the area of the corresponding triangle: $CS_0 = \frac{1}{2} \cdot 6 \cdot 30 = \90 . When the price increases to $p_1 = \$5$ per pound, the quantity demanded is $Q_1 = 50 - 5 \cdot 5 = 25$ and the consumer surplus is $CS_1 = \frac{1}{2} \cdot 5 \cdot 25 = \62.5 . Consumers then experience a loss of $\Delta CS = CS_1 - CS_0 = 62.5 - 90 = \27.5 . Hence, they will be willing to bribe Beth up to \$27.5 (the shaded area in the graph below) to prevent the restrictions from being imposed.



(iii) (2 points) Now suppose that Beth does not know the demand function, but she observed that when the price of oranges grew from \$4 to \$4.05 per pound, the quantity of oranges demanded fell from 30 to 29 pounds. What is the point elasticity of demand? Using this estimate, by how much would demand decrease if the government were to impose the quantity restrictions and raise the price from \$4 to \$5 (a 25% increase in the price)?

Answer: To answer this question, remember that the elasticity of demand shows the percentage change in demand due to a 1% change in price. So Beth first needs to calculate the (point) elasticity of demand using the information she already has (as in the lecture notes, I will use X to denote quantity demanded). The change in quantity demanded is $\Delta X = 29 - 30 = -1$ and the change in price is $\Delta p = 4.05 - 4 = 0.05$. The elasticity of demand is then

$$\epsilon = -\frac{\Delta\%X}{\Delta\%p} = -\frac{\Delta X}{\Delta p} \cdot \frac{p}{X} = -\frac{-1}{0.05} \cdot \frac{4}{30} = 2.67.$$

So, when price changes by 1%, quantity demanded changes by 2.67%. In the case of the quantity restrictions, the price increases by 25%. This implies that quantity demanded will fall by $2.67 \cdot 25 = 66.67\%$ to 10 pounds.

(iv) (2 points) After sitting in Econ306, Beth learned that you need to use the arc elasticity of demand for large price changes. She knows that, one year ago, when the price of oranges fell from \$4 to \$3 per pound, the quantity demanded increased from 30 to 50 pounds. What is her estimate of the arc elasticity? Using this estimate, what is the effect of the government restrictions on the demand for oranges?

Answer: For the arc elasticity, we need to calculate the changes in price and quantity demanded as well as the average price and quantity demanded:

$$\begin{aligned} \Delta X &= 50 - 30 = 20, & \bar{X} &= \frac{30 + 50}{2} = 40, \\ \Delta p &= 3 - 4 = -1, & \bar{p} &= \frac{4 + 3}{2} = 3.5. \end{aligned}$$

The arc elasticity of demand is then

$$\epsilon_a = -\frac{\Delta X}{\Delta p} \cdot \frac{\bar{p}}{\bar{X}} = -\frac{20}{-1} \cdot \frac{3.5}{40} = 1.75.$$

Therefore, if the price changes by 1%, quantity demanded changes by 1.75%. In the case of the quantity restrictions, when the price increases by 25%, quantity demanded will fall by $1.75 \cdot 25 = 43.75\%$ to 16.875 pounds.

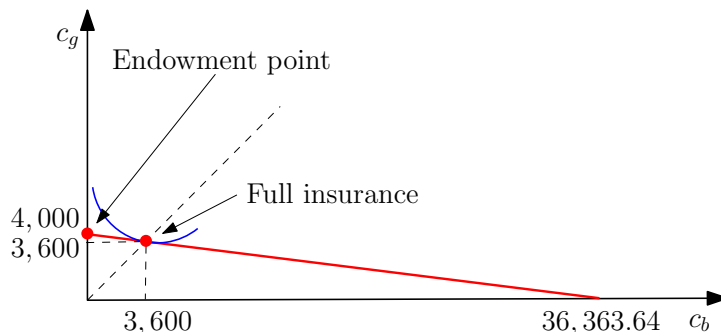
Question 5 (8 points)

Joanne has a beauty salon and she makes \$4,000 a month. Despite her uncontested talent, she tends to be absent minded at times. As a result, there is a 10% chance each

month that she will injure one of her customers and she will get sued. In that case, she will end up losing all her income that month. Since she is risk averse, she is looking into buying insurance.

(i) (4 points) What would be the premium of an actuarially fair insurance for Joanne? Sketch the budget constraint she would face, if she could buy the fair insurance, on a graph with consumption if not sued on the vertical axis and consumption if sued on the horizontal axis. Make sure to label the endowment point and to calculate the slope of the budget line. How much insurance will she buy?

Answer: The premium of an actuarially fair insurance is equal to the probability of the “bad” state, which in this case is the probability of Joanne being sued: $p = 10\%$. Thus, the premium is $r = \$0.10$, which means that the slope of the budget line faced by Joanne is $-\frac{r}{1-r} = -\frac{p}{1-p} = -\frac{0.1}{1-0.1} = -0.11$. This line has to pass through the endowment point, which is given by Joanne’s consumption if she does not buy insurance. In that case, her consumption if she does not get sued (the “good” case) is $c_g = \$4,000$, while her consumption if she gets sued (the “bad” case) is $c_b = \$0$. The budget line will then look like in the graph below.



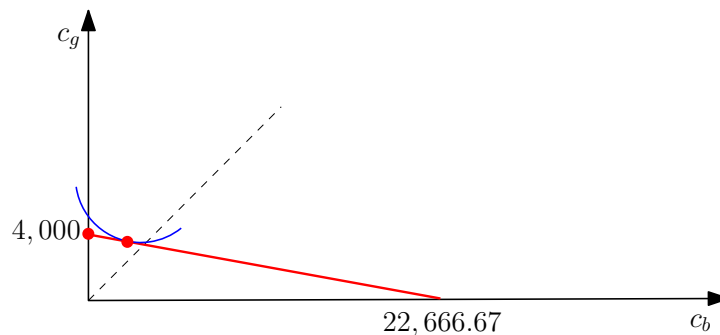
You can actually calculate the horizontal intercept and the full insurance consumption level. Since the vertical intercept is equal to 4000 and the slope of the line is -0.11 , the horizontal intercept is equal to $\frac{4000}{0.11} = 36,363.64$. As for the full insurance consumptions, remember that if Joanne buys x dollars of insurance, then she has to pay $r \cdot x$ in premiums regardless of whether she gets sued or not, but she also gets x dollars from the insurance company if she gets sued. Since full insurance means that consumption when sued equals consumption when not sued, it has to be that

$$4000 - r \cdot x = x - r \cdot x \quad \Rightarrow \quad x = 4,000.$$

So, whether she gets sued or not, Joanne will have $4000 - 0.1 \cdot 4000 = \$3,600$ left for consumption.

(ii) (4 points) Suppose that, instead, Joanne can only buy insurance at a \$0.15 premium. Draw the budget line that she faces (again, remember to label the endowment point and to calculate the slope of the budget line). Is this line flatter or steeper than the one in the previous part? How much insurance will she buy now? Compare your answer to the one in the previous part.

Answer: The slope of the budget line when Joanne faces actuarially unfair insurance is $-\frac{r}{1-r} = -\frac{0.15}{1-0.15} = -0.18$. This line has to pass through the endowment point again, so it will be a steeper line than in the previous case, starting from the same vertical intercept. We can calculate the horizontal intercept in the same way as before: $\frac{4000}{0.18} = 22,666.67$. The budget line will then look like in the graph below.

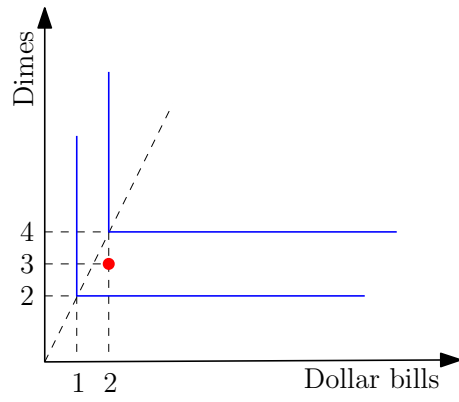


In this case, Joanne will *not* choose full insurance anymore, so her consumption bundle will lie to the left of the full insurance bundle (the intersection of the 45 degree line and the budget line).

Question 6 (extra 5 points)

Elmo finds himself at a soda vending machine on a hot and dusty Sunday. The machine requires exact change: one dollar bill and two dimes. No other combination will make anything come out of the machine. No stores are open and no one is in sight. Elmo is so thirsty that the only thing he cares about is how many sodas he can buy with the change in his pocket; the more he can buy, the better. From Elmo's point of view, preferences are now defined over dollar bills and dimes (and utility is derived from drinking the sodas that he can afford). What kind of goods are the dollar bills and dimes, in this particular situation? How many sodas can he buy with 2 dollar bills and 3 dimes? Draw an indifference curve describing Elmo's preferences.

Answer: Since Elmo needs exactly one dollar bill and two dimes to buy a soda and he has no other use for either dollar bills or dimes, the two goods are perfect complements. In this case, his indifference curves will look like in the graph below.



Finally, note that he will be able to buy only one soda with 2 dollar bills and 3 dimes, meaning that the indifference curves are rather “areas”: all the points between two indifference curves (given by an integer number of sodas he can buy) give him the same level of satisfaction as the points on the lower curve.