Econ306 – Intermediate Microeconomics
Handout on Price-Taking Firms

1 Profit maximization and the marginal output rule

A firm's profit is calculated as the difference between revenue and costs. When the firm is a price-taker, it does not think that its production decision will influence the price of its product or of the inputs (wage for labor and rent for capital). Then, the profit can be calculated as:

$$\pi = pQ - (wL + rK),$$

where Q is the amount of output, p the price of the product, L and K are the number of workers and the amount of capital used in the production process, and w and r are the wage and rent.

More generally, the cost of the firm will be expressed as a function of the output Q. (Remember that, in the short run, only the variable cost is an economic cost, meaning that only the variable cost will be taken into account in the calculation of the total cost in the profit. In the long run, since all costs are "variable", the total cost will just be the total cost of the firm.) The profit function can then be calculated as

$$\pi(Q) = pQ - TC(Q),$$

where $C(Q)$ is the total cost incurred by the firm when producing the quantity Q of output.

The objective of the firm is to maximize its profit. We can calculate the optimal level of production Q^* by taking the derivative of the profit function above and set it to zero:

$$\frac{\partial \pi(Q^*)}{\partial Q} = 0 \quad \Rightarrow \quad p - \frac{\partial TC(Q^*)}{\partial Q} \quad \Rightarrow \quad \frac{\partial TC(Q^*)}{\partial Q} = p \quad \Rightarrow \quad MC(Q^*) = p,$$

where the last equality follows from the definition of the marginal cost (remember again that in the short run the derivative of the total cost is equal to the derivative of the variable cost).

This last equation gives us the **marginal output rule**: *the optimal level of production Q^* is the level that sets the marginal cost equal to the price of the good produced.*

As a final note on this point, remember that the profit of a price-taking firm is equal to zero in the long run, because of free entry. In that case, the profit-maximizing output level is such that

$$\pi(Q^*) = 0 \quad \Rightarrow \quad pQ^* - (wL + rK) = 0 \quad \Rightarrow \quad pQ^* = wL + rK.$$

In other words, an output level that satisfies the equation above is a profit-maximizing level.

2 Short-run supply curve of the firm

Firms apply the marginal output rule to determine their optimal production level. What is special about the short run is that firms can change only the flexible factor(s), labor in our example, and as a result entry is somewhat limited (because of the time required to generate the capital for start-up).

It is easier to show the behavior of a firm using an example. Suppose that the variable cost of the firm is

$$VC(Q) = Q^3 - 8Q^2 + 20Q.$$

First, let us calculate the cost-minimizing output level $\overline{Q}$. This is the level at which average and marginal costs are equal. The marginal cost of the firm is

$$MC(Q) = \frac{\partial VC(Q)}{\partial Q} = 3Q^2 - 16Q + 20.$$

and the average cost is

$$AVC(Q) = \frac{VC(Q)}{Q} = \frac{Q^3 - 8Q^2 + 20Q}{Q} = Q^2 - 8Q + 20.$$

The cost-minimizing level of output is then given by:

$$MC(\overline{Q}) = AVC(\overline{Q}) \quad \Rightarrow \quad 3\overline{Q}^2 - 16\overline{Q} + 20 = \overline{Q}^2 - 8\overline{Q} + 20 \quad \Rightarrow \quad 2\overline{Q}^2 - 8\overline{Q} = 0 \quad \Rightarrow$$
$$2\overline{Q}(\overline{Q} - 4) = 0 \quad \Rightarrow \quad \overline{Q} = 4.$$

At this level, we can calculate the following quantities:

$$\begin{aligned} p^* &= MC(\overline{Q}) = AVC(\overline{Q}) = 3\overline{Q}^2 - 16\overline{Q} + 20 = 3 \cdot 16 - 16 \cdot 4 + 20 = 4, \\ VC(\overline{Q}) &= AVC(\overline{Q}) \cdot \overline{Q} = 4 \cdot 4 = 16, \\ \pi(\overline{Q}) &= p \cdot \overline{Q} - VC(\overline{Q}) = 4p - 16, \end{aligned}$$

where p^* is the price which is equal to the marginal and the average cost, and the formula for the profit function is due to the fact that the only economic cost in the short run is the variable cost.

Now suppose that the market price of the product is $p_1 = \$15$ (which is greater than p^*). Then, the optimal output level Q_1^* can be determined according to the marginal output rule:

$$MC(Q_1^*) = p_1 \quad \Rightarrow \quad 3Q_1^{*2} - 16Q_1^* + 20 = 15 \quad \Rightarrow \quad 3Q_1^{*2} - 16Q_1^* + 5 = 0 \quad \Rightarrow$$
$$Q_1^* = \frac{16 \pm \sqrt{16^2 - 4 \cdot 3 \cdot 5}}{2 \cdot 3} = \frac{16 \pm \sqrt{196}}{6} = \frac{16 \pm 14}{6}.$$

Ignoring the solution that is almost equal to zero, this gives us the profit-maximizing level of output:

$$Q_1^* = \frac{16 + 14}{6} = \frac{30}{6} = 5 > \overline{Q}.$$

The profit of the firm is now:

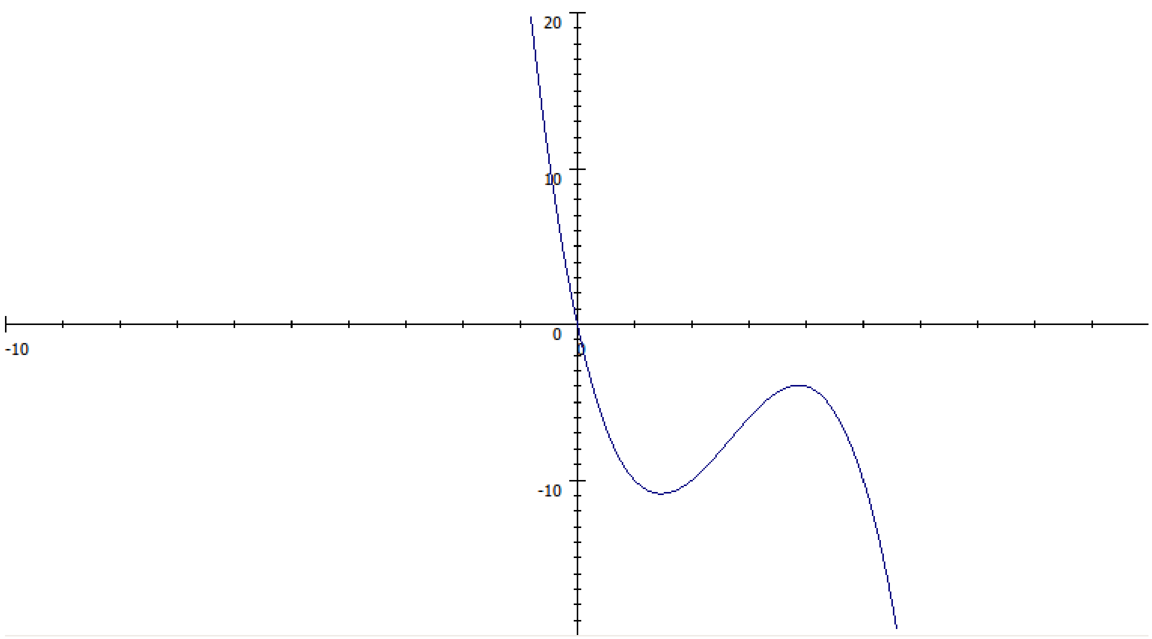
$$\pi(Q_1^*) = 15 \cdot 5 - VC(Q_1^*) = 75 - \left(5^3 - 8 \cdot 5^2 + 20 \cdot 5\right) = 75 - 25 = 50.$$

Since the firm makes a positive profit, it will choose to produce and supply a quantity of 5 units of its product. The same will hold for every price above p^*: the firm will make profit and thus will choose to produce a quantity higher than $\overline{Q}$, as given by the equation that sets the marginal cost equal to the market price.

Now suppose that the market price is $p_2 = \$3$ (which is lower than p^*). The lowest level of the average cost (which is attained when the average cost equals the marginal cost) is $AVC(\overline{Q}) = \$4$, as calculated above. This means that at every production level the marginal revenue (price) is lower than the average cost. Then, according to the shutdown rule, the firm should not produce—it would run losses, regardless of how much it chooses to produce. To see this, let us calculate the profit function:

$$\pi(Q) = 3Q - \left(Q^3 - 8Q^2 + 20Q\right) = -Q^3 + 8Q^2 - 17Q.$$

The graph of the profit function is shown below. As you can see, the profit is negative for any positive production level, meaning the firm is better off going out of business.



Finally, let us see what happens if the market price of the product is equal to $p^* = \$4$. In this case, the firm will choose to produce a quantity $\overline{Q} = 4$ and its profit is (using the values calculated before)

$$\pi(\overline{Q}) = 4 \cdot 4 - 16 = 0.$$

Hence, the firm is indifferent between producing and being out of business (in which case we'll assume that the firm chooses to produce).

In conclusion, we derived the short-run supply curve of the firm:

- if $p > p^*$, then produce at the level given by the marginal output rule: $MC(Q) = p$;
- if $p < p^*$, then go out of business;
- if $p = p^*$, the choose the cost-minimizing output level and earn zero profits.