Chapter 4

Understanding Interest Rates

Present Value

- Concept
  - a dollar today is (in general) more valuable than a dollar tomorrow – you can invest this dollar (e.g., deposit it in a bank) so that tomorrow you have more than $1
- Example: Simple loan of $1 at 10% interest

Year	1	2	3	…	n
	$1.10	$1.21	$1.33		$1x(1 + i)^n

$$\text{PV of future } \$x = \frac{x}{(1+i)^n}$$

This is called *discounting the future*

Present Value – Example 2

- suppose you won the lottery and are promised \$20 million in payments of \$1 million over the next 20 years
- what you get is actually less than \$20million, because the present value of these payments is

$$\text{PV} = \frac{\$1mil}{(1+i)} + \frac{\$1mil}{(1+i)^2} + \ldots + \frac{\$1mil}{(1+i)^{20}}$$

- if we assume that i = 10%, the present value of your lottery prize is actually \$9.4 million!

Credit Instruments

- Loans
  - *simple loan* = the borrower must repay the funds loaned at maturity date, along with an additional payment for the interest (e.g., commercial loans to businesses)
  - *fixed-payment* (*fully amortized*) *loan* = the borrower makes the same payment every period (principal plus interest) for a set number of years (e.g., auto loans, mortgages)

Credit Instruments (cont.)

- Bonds
  - *coupon bond* = the owner (i.e., lender) receives a fixed interest payment (called *coupon payment*) every period until maturity, when the final amount (called *face value* or *par value*) is repaid (e.g., Treasury bonds, corporate bonds)
  - *discount* (*zero coupon*) *bond* = bought at a price below its face value, while the face value is repaid at maturity (no interest payments)

Yield to Maturity

- *yield to maturity* (i) = interest rate that equates today's value with present value of all future payments

- Loans:
  - simple loan, $100 for 1 year, 10% interest rate:

$$\$100 = \frac{\$110}{(1+i)} \Rightarrow i = \frac{\$110 - \$100}{\$100} = 10\% = \text{interest rate}$$

  - fixed-payment loan – more complicated:

$$\text{Loan value} = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + ... + \frac{FP}{(1+i)^n}$$

Yield to Maturity

- Coupon bond:
  - same strategy as fixed-payment loan, using price P as value of loan

    $$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + ... + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

    where F is the face value of the bond and

    $C = r \times F$ is the yearly coupon payment (r is the coupon rate)
  - for a *consol* (*perpetuity*), i.e. a coupon bond with no maturity date and no payment of principal,

    $$P = \frac{C}{i} \Rightarrow i = \frac{C}{P}$$

Yield to Maturity (cont.)

- Discount bond:
  - same strategy as simple loan, using price as value of loan

$$P = \frac{F}{(1+i)^n}$$

  - for a one-year discount bond,

$$P = \frac{F}{1+i} \Rightarrow i = \frac{F-P}{P}$$

Yield to Maturity (cont.)

- note that in the case of bonds, the yield to maturity is not necessarily equal to coupon rate
- also, for both consols and discount bonds, the yield to maturity is negatively related to price
- it can be shown that this relationship holds also for general coupon bonds
- hence, current bond prices and interest rates are negatively related
- when the bond is at par (i.e., price = face value), the yield to maturity and coupon rate are equal

Relationship Between Price and Yield to Maturity

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

Three Interesting Facts in Table 1

1. When bond is at par, yield equals coupon rate
2. Price and yield are negatively related
3. Yield greater than coupon rate when bond price is below par value

Current Yield

- yield to maturity is the most accurate measure of interest rates (and it is what economists mean by "interest rates")
- yield to maturity is difficult to calculate – other useful approximations are easier
- *current yield* is calculated for coupon bonds:

$$i_c = \frac{C}{P} \Rightarrow \text{ negatively related to price}$$

- it is a better approximation to yield to maturity, the nearer price is to par and the longer is the maturity of the bond
- the change in current yield always signals change in same direction as yield to maturity

Yield on a Discount Basis

- is calculated for discount bonds:

$$i_{db} = \frac{F - P}{F} \times \frac{360}{\text{days to maturity}}$$

- again this is negatively related to price
- it understates yield to maturity – the longer the maturity, the greater is the understatement
- the change in discount yield always signals a change in the same direction for the yield to maturity

Distinction Between Interest Rates and Returns

- the interest rate (yield to maturity) is *not* the same as the rate of return from buying the bond
- *rate of return* = total net benefit from investment (payments to the owner plus change in value) divided by the initial cost
- the rate of return for a bond held from t to $t+1$ is

$$R = \frac{P_{t+1} - P_t + C}{P_t} = \frac{P_{t+1} - P_t}{P_t} + \frac{C}{P_t} = g + i_c$$

where g is the rate of capital gain

Table 2 Analysis

- table 2 looks at the one-year return on several 10%-coupon-rate bonds purchased at par
- the interest rate (yield to maturity) changes from 10% to 20% during the year
- the “price next year” is calculated using the formula of the yield to maturity for coupon bonds

- Conclusions:
  - prices and returns are more volatile for long-term bonds because they have higher interest-rate risk
  - there is no interest-rate risk for any bond whose maturity equals holding period

Key Facts about the Relationship between Interest Rates and Returns

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated using Equation 3.

Key Findings from Table 2

- the only bond whose return equals the yield is the one with maturity equal to the holding period
- for bonds with maturity longer than the holding period, $i\uparrow$, $P\downarrow$, implying a capital loss
- the longer is maturity, the greater is the percentage price change associated with the interest rate change
- the longer is maturity, the more return changes with change in the interest rate
- a bond with high initial interest rate can still have a negative return if $i\uparrow$

Real Interest Rate

- *real interest rate* = interest rate that is adjusted for expected changes in the price level (i.e., expected inflation, π^e)

  $i_r = i - \pi^e$

- the real interest rate reflects more accurately the true cost of borrowing
- when the real rate is low, there are greater incentives to borrow and less to lend
- it can be (and sometimes is) negative, if expected inflation is higher than the nominal interest rate

U.S. Real and Nominal Interest Rates