

Risk and Risk Aversion

Chapter 6

Why the Need for a New Theory?

- economic decisions under uncertainty are not based only on monetary outcomes
- St. Petersburg Paradox (Bernoulli, 1738)
 - a coin is tossed until "head" appears (toss n)
 - payoff from participating: $R(n) = 2^n$
 - how much would you pay as entry fee?
- people usually exhibit *decreasing marginal utility* (e.g., log utility) → risk aversion

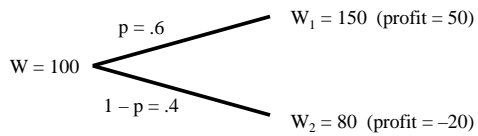
1-2

Risky Investments

- Lotteries
 - *simple lotteries* = investment opportunities where a certain wealth is put at risk and only two outcomes are possible
 - *compound lotteries* allow for more than two outcomes and can be interpreted as combinations of simple lotteries
- elements of a lottery:
 - final wealth for each possible outcome
 - probabilities associated to each possible outcome

1-3

Risk - Uncertain Outcomes



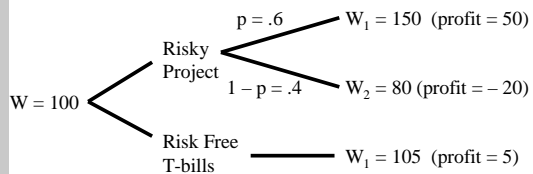
$$E(W) = pW_1 + (1 - p)W_2 = .6(150) + .4(80) = 122$$

$$\sigma^2 = p[W_1 - E(W)]^2 + (1 - p)[W_2 - E(W)]^2 = .6(150 - 122)^2 + .4(80 - 122)^2 = 1,176$$

$$\sigma = 34.293$$

1-4

Risky Investments with Risk-Free



$$\text{Risk Premium} = E(W) - \text{Risk-free return} = 17$$

1-5

Risk Aversion

- *fair game* = lottery with zero risk premium
- investor's view of risk
 - *risk averse* = reject investment projects that are fair games or worse
 - require a risk premium
 - risk premium increases with risk
 - *risk neutral* = evaluate investment projects based only on expected returns (ignore risk)
 - *risk lover* = prefer higher risk (similar to requiring a *negative* risk premium)
 - most individuals are risk averse

1-6

Risk Aversion & Utility

■ Utility Function

- *mean-variance criterion* = individuals compare investment opportunities based on expected return and risk (variance)
- example of a *utility function* for risk averse individuals:
$$U = E(r) - 0.005 A \sigma^2$$
- A measures the degree of risk aversion (higher A corresponds to more risk-averse individuals)
- risk aversion: U increases with $E(r)$ and falls with σ^2

1-7

Risk Aversion & Utility (cont.)

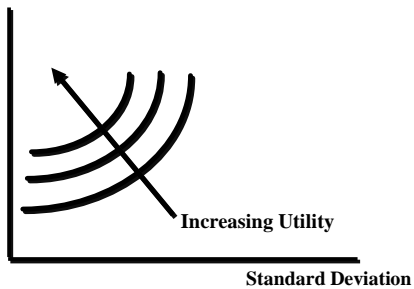
■ mean-variance criterion

- *certainty equivalent (rate)* = risk-free rate that gives the same utility as the risky portfolio
- an individual always rejects an investment portfolio with certainty equivalent rate less than the risk-free rate
- *dominance principle* = investment A dominates investment B if it offers higher expected return and lower risk, at least one strictly
- *indifference curve* = set of investment opportunities that give the same utility

1-8

Indifference Curves

Expected Return



1-9

Asset Risk vs. Portfolio Risk

- investment projects or *portfolios* are composed of many different assets
- *hedging* = investing in an asset that tends to offset exposure to a certain kind of risk
- *diversification* = strategy based on investing in a variety of assets so that exposure to any kind of particular risk is limited

1-10

Asset Expected Return and Variance

- *expected return* of an asset = probability weighted average return in all scenarios

$$E(r) = \sum_s P(s)r(s)$$

- *variance* of an asset's return = expected value of the squared deviations from the expected return

$$\sigma^2 = \sum_s P(s)[r(s) - E(r)]^2$$

1-11

Return on a Portfolio

- *rate of return on a portfolio* = weighted average of the rates of return of each asset comprising the portfolio, with the portfolio proportions as weights

$$r_p = w_1 r_1 + w_2 r_2$$

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2)$$

where:

w_1 = proportion of funds in security 1

w_2 = proportion of funds in security 2

r_1 = expected return on security 1

r_2 = expected return on security 2

1-12

Portfolio Risk

- when a risky asset is combined with a risk-free asset, the portfolio standard deviation is

$$\sigma_p = w_{\text{riskyasset}} \times \sigma_{\text{riskyasset}}$$

$$\sigma_p^2 = w_{\text{riskyasset}}^2 \times \sigma_{\text{riskyasset}}^2$$

- when two risky assets with variances σ_1^2 and σ_2^2 , respectively, are combined into a portfolio with portfolio weights w_1 and w_2 , respectively, the portfolio variance is:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}(r_1, r_2)$$
