Risk and Risk Aversion

Chapter 6

Why the Need for a New Theory?

- economic decisions under uncertainty are not based only on monetary outcomes
- St. Petersburg Paradox (Bernoulli, 1738)
  - a coin is tossed until "head" appears (toss n)
  - payoff from participating: $R(n) = 2^n$
  - how much would you pay as entry fee?
- people usually exhibit *decreasing marginal utility* (e.g., log utility) ➔ risk aversion

Risky Investments

- Lotteries
  - *simple lotteries* = investment opportunities where a certain wealth is put at risk and only two outcomes are possible
  - *compound lotteries* allow for more than two outcomes and can be interpreted as combinations of simple lotteries
  - elements of a lottery:
    - final wealth for each possible outcome
    - probabilities associated to each possible outcome

Risk - Uncertain Outcomes

W = 100

p = .6 → W_1 = 150 (profit = 50)

1 – p = .4 → W_2 = 80 (profit = –20)

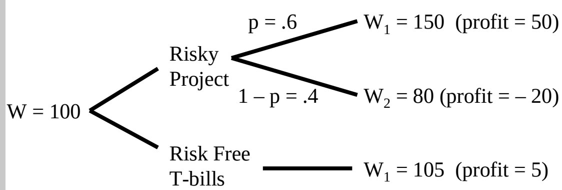
$$E(W) = pW_1 + (1-p)W_2 = .6\,(150) + .4(80) = 122$$

$$\sigma^2 = p[W_1 - E(W)]^2 + (1-p)\,[W_2 - E(W)]^2 =$$

$$.6\,(150-122)^2 + .4(80-122)^2 = 1{,}176$$

$$\sigma = 34.293$$

Risky Investments with Risk-Free



Risk Premium = E(W) – Risk-free return = 17

Risk Aversion

- *fair game* = lottery with zero risk premium
- investor's view of risk
  - *risk averse* = reject investment projects that are fair games or worse
    - require a risk premium
    - risk premium increases with risk
  - *risk neutral* = evaluate investment projects based only on expected returns (ignore risk)
  - *risk lover* = prefer higher risk (similar to requiring a *negative* risk premium)
  - most individuals are risk averse

Risk Aversion & Utility

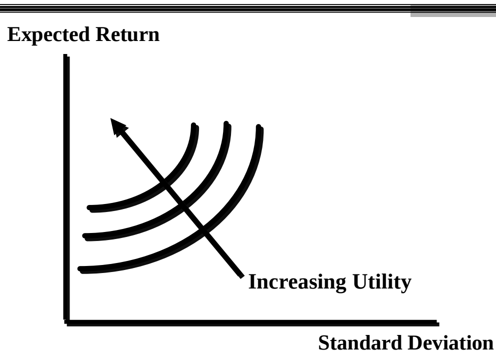
- Utility Function
  - *mean-variance criterion* = individuals compare investment opportunities based on expected return and risk (variance)
  - example of a *utility function* for risk averse individuals:

    $U = E(r) - 0.005\, A\, \sigma^2$
  - A measures the degree of risk aversion (higher A corresponds to more risk-averse individuals)
  - risk aversion: U increases with E(r) and falls with σ^2

Risk Aversion & Utility (cont.)

- mean-variance criterion
  - *certainty equivalent (rate)* = risk-free rate that gives the same utility as the risky portfolio
  - an individual always rejects an investment portfolio with certainty equivalent rate less than the risk-free rate
  - *dominance principle* = investment *A* dominates investment *B* if it offers higher expected return and lower risk, at least one strictly
  - *indifference curve* = set of investment opportunities that give the same utility

Indifference Curves



Asset Risk vs. Portfolio Risk

- investment projects or *portfolios* are composed of many different assets
- *hedging* = investing in an asset that tends to offset exposure to a certain kind of risk
- *diversification* = strategy based on investing in a variety of assets so that exposure to any kind of particular risk is limited

Asset Expected Return and Variance

- *expected return* of an asset = probability weighted average return in all scenarios

$$E(r) = \sum_s P(s) r(s)$$

- *variance* of an asset's return = expected value of the squared deviations from the expected return

$$\sigma^2 = \sum_s P(s)[r(s) - E(r)]^2$$

Return on a Portfolio

- *rate of return on a portfolio* = weighted average of the rates of return of each asset comprising the portfolio, with the portfolio proportions as weights

$$r_p = w_1 r_1 + w_2 r_2$$
$$\mathrm{E}(r_p) = w_1 \mathrm{E}(r_1) + w_2 \mathrm{E}(r_2)$$

where:

w_1 = proportion of funds in security 1
w_2 = proportion of funds in security 2
r_1 = expected return on security 1
r_2 = expected return on security 2

Portfolio Risk

- when a risky asset is combined with a risk-free asset, the portfolio standard deviation is

$$\sigma_p = w_{riskyasset} \times \sigma_{riskyasset}$$

$$\sigma_p^2 = w_{riskyasset}^2 \times \sigma_{riskyasset}^2$$

- when two risky assets with variances σ_1^2 and σ_2^2, respectively, are combined into a portfolio with portfolio weights w_1 and w_2, respectively, the portfolio variance is:

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov(r_1, r_2)$$