The Capital Asset Pricing Model

Chapter 9

Capital Asset Pricing Model (CAPM)

- centerpiece of modern finance
- gives the relationship that should be observed between risk and return of an asset
- it allows for the evaluation of:
  - future prices of stocks
  - “fair” price of stocks not trading yet (IPOs)
- derived using principles of diversification with simplified assumptions.

Assumptions

- individual investors are price takers = they act as if their actions do not affect prices (perfect competition)
- single-period investment horizon = all investors plan to hold assets for the same period (myopic behavior)
- investments are limited to traded financial assets – rules out investment in human capital and borrowing restrictions
- no taxes and transaction costs = no fees or commissions, or income taxes

Assumptions (cont.)

- investors are rational mean-variance optimizers = they use the Markowitz portfolio selection model
- information is costless and available to all investors
- there are *homogeneous expectations* = all investors share the same view of the world (i.e., they derive the same efficient portfolio frontier)

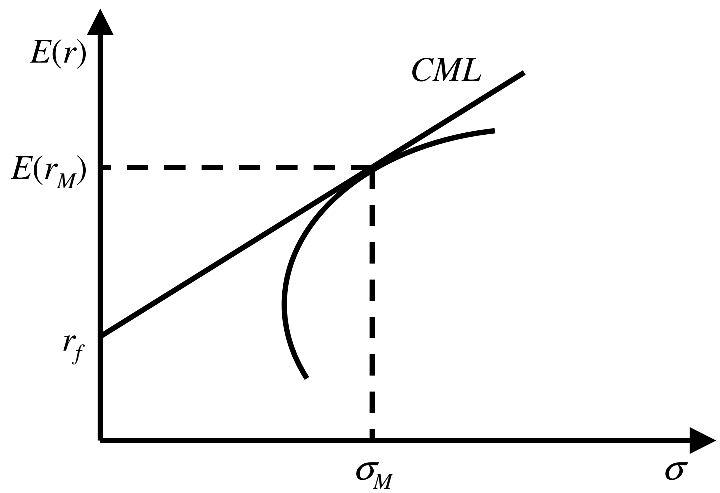
Optimal Risky Portfolio

- since all investors have the same portfolio frontier, all investors will hold the same portfolio for risky assets (the only difference is amount invested in it vs. the risk-free asset)
- sum up the holdings of all investors in the market: borrowing and lending cancel out ➔ net wealth of the economy
- all individuals hold the same risky portfolio ➔ proportion in portfolio = proportion in the market

Resulting Equilibrium Condition

- All investors will hold the same portfolio for risky assets, the *market portfolio*
  - market portfolio = contains all securities and the proportion of each security is its market value (price times number of shares) as a percentage of total market value
  - market portfolio is not only efficient, it is also the tangency point to the optimal CAL
  - the Capital Market Line becomes the best attainable CAL

Capital Market Line



Market Portfolio

- the fact that all assets are included and that the proportions in the individual risky portfolio and in the market portfolio are equal are ensured by the pricing mechanism
- *mutual fund theorem*: passive strategy of investing in the market index is efficient
- another form of the separation property:
  - broker finds the market portfolio (the optimal risky portfolio)
  - investors decide how much to invest in the market portfolio versus the risk-free asset

Risk Premium on the Market Portfolio

- depends on the "average" degree of risk aversion (i.e., the degree of risk aversion of a typical investor)
- recall that

$$y = \frac{E(r_p) - r_f}{0.01 A \sigma_p^2}$$

- since borrowing and lending offset in the aggregate, $y = 1$
- rearranging:

$$E(r_M) - r_f = 0.01 \overline{A} \sigma_M^2$$

Risk and Return of Individual Securities

- what matters is not individual security risk, but *portfolio risk* → when assessing a security, what matters is its *contribution* to portfolio risk
- portfolio risk with many assets:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i, r_j)$$

- the contribution of stock k to the portfolio:

$$\sum_{j=1}^{n} w_k w_j Cov(r_k, r_j) = w_k Cov(r_k, r_p)$$

- similarly, stock k's contribution to the risk premium of the portfolio:

$$w_k \, [E(r_p) - r_f]$$

Market Risk and Return

- so, when compared to market portfolio:

  Contribution to return = $w_k\ [E(r_M) - r_f]$

  Contribution to risk = $w_k\ Cov(r_k, r_M)$
- hence, *reward-to-risk ratio* is:

$$\text{Reward} - \text{to} - \text{risk ratio} = \frac{E(r_k) - r_f}{Cov(r_k, r_M)}$$

- the reward-to-risk ratio of the market portfolio is called the *market price of risk*:

$$\text{Market price of risk} = \frac{E(r_M) - r_f}{\sigma_M^2}$$

Equilibrium Reward-to-Risk Ratio

- if an investment has lower reward-to-risk ratio than another (or the average), then investors would move away from it → price falls → return increases → reward-to-risk ratio rises
- conversely, if an investment has a higher reward-to-risk ratio than another (or the average), then investors would tilt toward it → price rises → return falls → reward-to-risk ratio decreases
- in equilibrium, all investments should offer the same reward-to-risk ratio

Equilibrium Risk-Return Relationship

- hence, any stock should have the same reward-to-risk ratio as the market portfolio:

$$\frac{E(r_k)-r_f}{Cov(r_k,r_M)}=\frac{E(r_m)-r_f}{\sigma_M^2}$$

- rearranging, this gives the equilibrium risk-return relationship:

$$E(r_k)-r_f=\frac{Cov(r_k,r_M)}{\sigma_M^2}\left[E(r_m)-r_f\right]$$

Beta

- the ratio of the contribution of stock k to the market portfolio risk to total risk is called *beta*:

$$\beta_k=\frac{Cov(r_k,r_M)}{\sigma_M^2}$$

- the usual expression of the *capital asset pricing model* (*CAPM*) is the relationship between expected return and beta:

$$E(r_k) = r_f + \beta_k\,[E(r_M) - r_f]$$

- hence, the right measure of risk (in this framework) is *beta*

Portfolio Beta

- if the expected return-beta relationship holds for any individual asset, it has to hold for any combination of assets as well:

$$
\begin{array}{l}
w_1 E(r_1) = w_1 r_f + w_1 \beta_1 \left[E(r_M) - r_f \right] \\
+ w_2 E(r_2) = w_2 r_f + w_2 \beta_2 \left[E(r_M) - r_f \right] \\
\qquad \vdots \\
+ w_n E(r_n) = w_n r_f + w_n \beta_n \left[E(r_M) - r_f \right] \\
\hline
E(r_p) = r_f + \beta_p \left[E(r_M) - r_f \right]
\end{array}
$$

where the *portfolio beta* is

$$\beta_p = w_1\beta_1 + w_2\beta_2 + \ldots + w_n\beta_n$$

Intuition

- beta is proportional to the contribution of an asset to the risk of the optimal risky portfolio
- hence, beta is the “right” measure of risk
- risk-averse investors evaluate assets based on their risk ➔ risk premium should be a function of the right measure of risk
- this is the CAPM: the risk premium of an asset is proportional to its beta

Cautions

- important distinction between *firm return* (as measured by dividends, etc.) and *stock returns* (as measured by the rate of return on holding stocks)
- if everybody expects a company to do well and pay large dividends (as information is public), then price increases and expected return stays the same
- only the risk of the company (beta) influences expected returns

Security Market Line

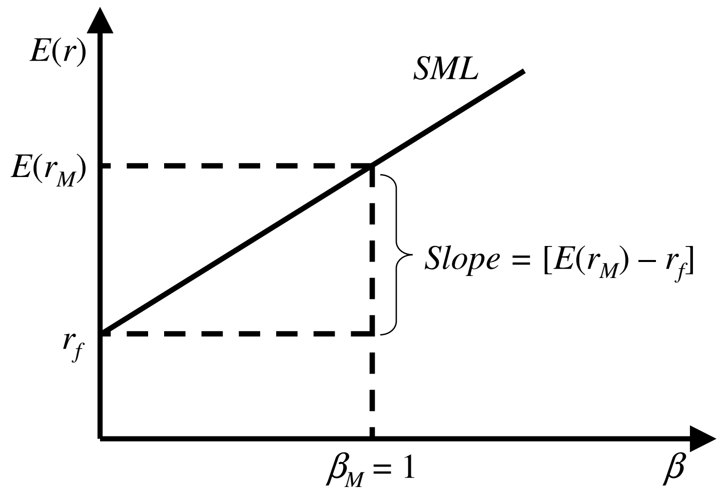
- the expected return-beta relationship can be interpreted as a line (in the expected return-beta plane), called *Security Market Line* (*SML*)
- slope of the SML = $[E(r_M) - r_f]$
- beta of market portfolio is

$$\beta_M = \frac{Cov(r_M, r_M)}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2} = 1$$

- beta of the risk-free asset is

$$\beta_f = \frac{Cov(r_f, r_M)}{\sigma_M^2} = \frac{0}{\sigma_M^2} = 0$$

Security Market Line



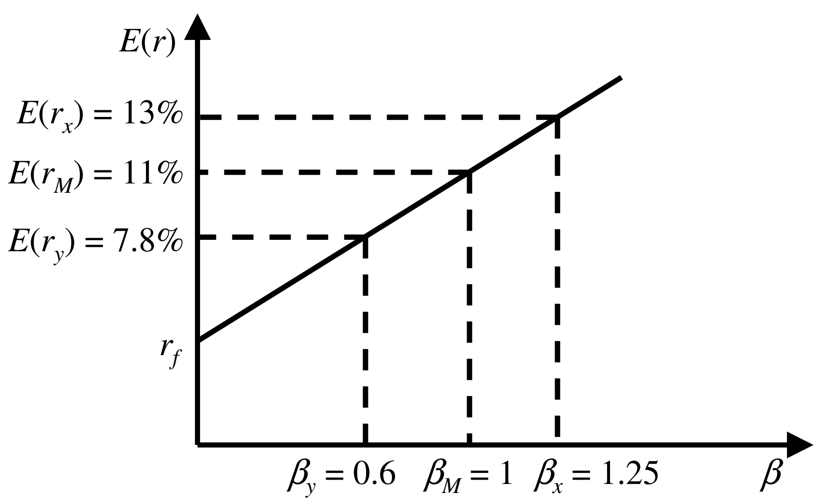
Security Market Line vs. Capital Market Line

- What is plotted
  - CML plots *efficient portfolios*, i.e. combinations of the risky portfolio and the risk-free asset (it is *not* valid for individual assets)
  - SML plots *individual assets and portfolios*
- Measure of risk
  - for CML – standard deviation (because well-diversified portfolios)
  - for SML – beta (because individual assets)

Example of SML

- $E(r_M) = 11\%$
- $r_f = 3\%$
- Market risk premium = $E(r_M) - r_f = 11 - 3 = 8\%$
- $\beta_x = 1.25$
- $E(r_x) = r_f + \beta_x [E(r_M) - r_f] = 3 + 1.25 \cdot 8 = 13\%$
- $\beta_y = 0.6$
- $E(r_y) = r_f + \beta_y [E(r_M) - r_f] = 3 + 0.6 \cdot 8 = 7.8\%$

Security Market Line



Usefulness of CAPM – Stock Pricing

- SML gives the "fair return" (and hence price) of a stock, given its risk (beta)
- in practice, assets might not lie exactly on the SML because of "pricing errors"
- an underpriced asset would give a higher expected return than predicted by SML, hence it would be plotted *above* the line
- conversely, an overpriced asset gives a lower expected return than predicted by the SML and would plot *below* the SML

Alpha

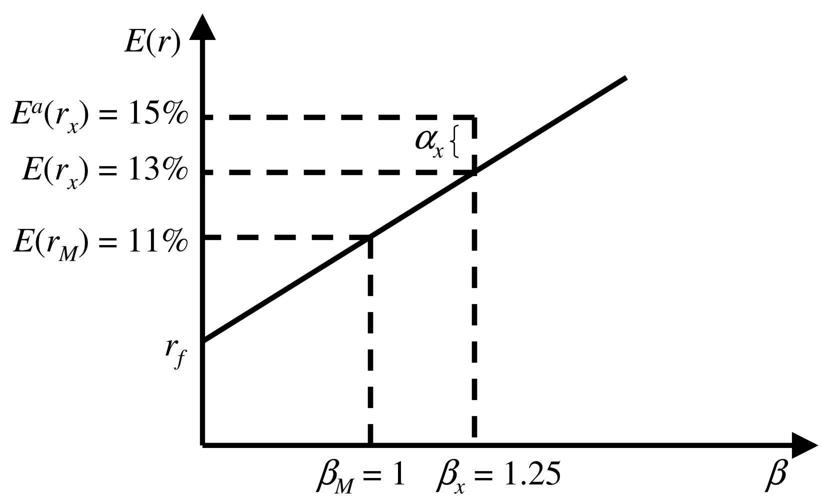
- the difference between the actual and the fair expected rates of return on an asset is called *alpha*:

  $\alpha = E^a(r) - E(r)$
- if $\alpha > 0$, the stock is underpriced (hence desirable to invest in)
- if $\alpha < 0$, the stock is overpriced (hence undesirable to invest in)

Example of Calculating Alphas

- $E(r_M) = 11\%$
- $r_f = 3\%$
- Market risk premium = $E(r_M) - r_f = 11 - 3 = 8\%$
- $\beta_x = 1.25$
- $E^a(r_x) = 15\%$
- $E(r_x) = r_f + \beta_x [E(r_M) - r_f] = 3 + 1.25[11 - 3] = 13\%$
- $\alpha_y = E^a(r_x) - E(r_x) = 15 - 13 = 2\%$
- hence, stock X is underpriced

Security Market Line



Usefulness of CAPM – Budgeting Decisions

- CAPM gives the required rate of return of an investment project, given its risk, so that investors find it acceptable
- managers can use CAPM to find the cutoff *internal rate of return*

Extensions of the CAPM – No Borrowing

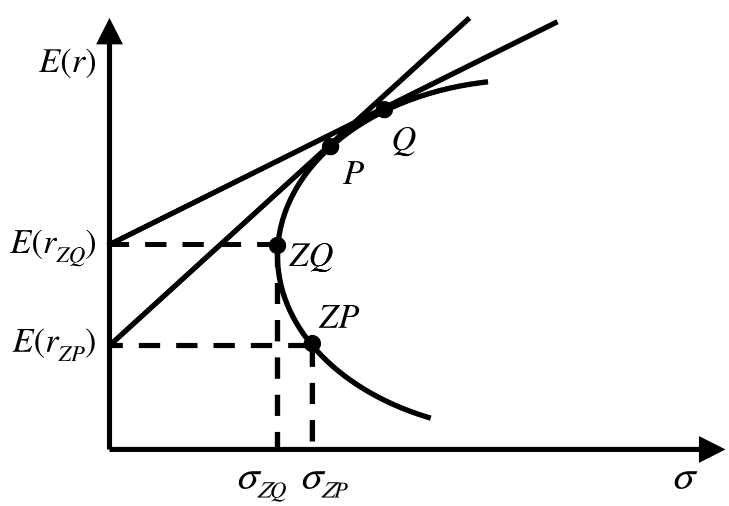
- CAPM is based on the separation principle: all investors find the same risky portfolio to be optimal
- when borrowing is restricted, the separation principle fails ➔ market portfolio is not the common optimal risky portfolio anymore
- hence, CAPM fails
- Black (1972) provides a model that extends the CAPM to cases where borrowing is partially or completely restricted (i.e., no risk-free asset)

Zero-Beta Model

- Implications:
  - any combination (i.e., portfolio) of efficient portfolios is also efficient
  - for every efficient portfolio there is an inefficient portfolio on the mean-variance frontier with which it is uncorrelated (the *zero-beta portfolio*)
  - the expected return of any asset can be found using any two frontier portfolios *P* and *Q*:

$$E(r_k) = E(r_Q) + \frac{Cov(r_k, r_P) - Cov(r_P, r_Q)}{\sigma_P^2 - Cov(r_P, r_Q)}\left[E(r_P) - E(r_Q)\right]$$

Finding Zero-Beta Portfolios



Example – No risk-free asset

- suppose there is no risk-free asset
- then investors cannot borrow or lend at the risk-free rate
- investors will want to invest in efficient portfolios
- since any portfolio can be written as a combination of 2 frontier portfolios, any portfolio the investors choose can be written as a combination of the market portfolio and its zero-beta counterpart

Example – No risk-free asset (cont.)

- in this case, the equation determining the expected return of an asset becomes:

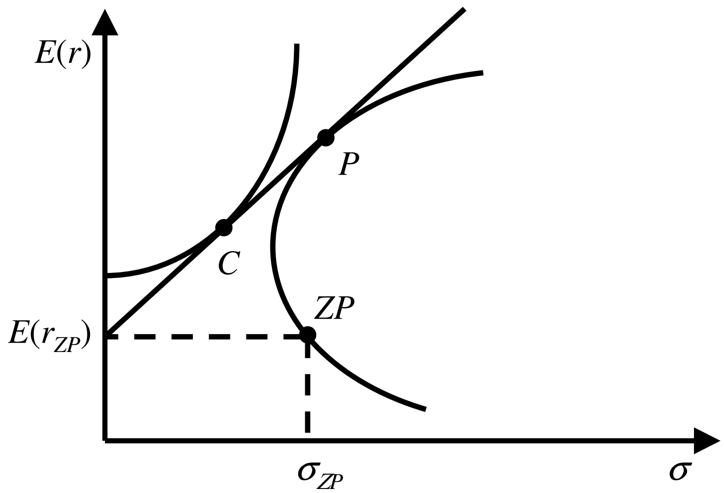
$$E(r_k) = E(r_{ZM}) + \frac{Cov(r_k, r_M) - Cov(r_M, r_{ZM})}{\sigma_M^2 - Cov(r_M, r_{ZM})}\left[E(r_M) - E(r_{ZM})\right]$$

$$= E(r_{ZM}) + \frac{Cov(r_k, r_M)}{\sigma_M^2}\left[E(r_M) - E(r_{ZM})\right]$$

$$= E(r_{ZM}) + \beta_k\left[E(r_M) - E(r_{ZM})\right]$$

- this is like the CAPM equation, but with $E(r_{ZM})$ instead of r_f

Zero-Beta Model – No Risk-free Asset



CAPM and Lifetime Consumption

- another extension concerns the time horizon investors consider
- investors may not wish to hold assets for just one period, or may have different holding periods
- Fama (1970) showed that the single-period CAPM is appropriate even in a multiperiod setting, under certain assumptions

CAPM and Liquidity

- yet another extension relates to the liquidity premium
- *liquidity* = the cost and ease with which an asset can be converted into cash (i.e., sold)
- researchers found that liquidity risk (i.e., the risk of not being able to sell rapidly and cheaply the asset) is systematic, hence difficult to diversify
- less liquid asset should offer a *liquidity premium* over more liquid assets

CAPM and Liquidity (cont.)

- hence, need to modify the CAPM framework
- since illiquid assets offer a liquidity premium, in the long run they offer higher rates of returns than their liquid counterparts
- the risk premium will take liquidity into account:

  $$E(r_k) - r_f = \beta_k [E(r_M) - r_f] + f(c_k)$$

  where $f(c_k)$ is the liquidity premium as a function of the transaction costs of asset k
- $f(c_k)$ is increasing in c_k, but at a decreasing rate
- a useful measure of liquidity is the bid-ask spread: more liquid assets have lower spreads

Liquidity and Average Returns