Arbitrage Pricing Theory and Multifactor Models of Risk and Return

Chapter 11

Single-Factor Model

- remember the single-factor model:

  $$r_i = E(r_i) + \beta_i F + e_i$$

  where F and e_i have zero mean (as they capture surprise changes in the factors)
- this allows the decomposition of risk into *market* and *firm-specific* components
- however, this decomposition is overly simplistic:
  - ignores certain factors (e.g., industry-specific)
  - assumes all stocks respond the same to common factors embodied in F

Multifactor Models

- we should allow for different stocks to have different sensitivities to different types of market-wide shocks (e.g., inflation, business cycles, interest rates etc.)
- *multifactor models* = models that allow for different sensitivities to different factors
- they can provide a better description of security returns

Example – A Two-Factor Model

- suppose we believe the only macroeconomic sources of risk are business cycles (GDP) and interest rates fluctuations (IR)
- rates of return should then respond to unanticipated changes in both factors:

$$r_i = E(r_i) + \beta_{i\,GDP}\,GDP + \beta_{i\,IR}\,IR + e_i$$

- the beta coefficients are called *factor sensitivities*, *factor loadings* or *factor betas*
- example: compare the sensitivity of returns on an utility company and an airline company

Determining $E(r_i)$

- our equation is just a *description* of returns, there is really no theory behind it
- where does $E(r)$ come from?
- we need a theory of market equilibrium
- CAPM is a theory of market equilibrium, but it only values *aggregated* market risk

$$E(r_i) = r_f + \beta_i [E(r_m) - r_f]$$

or, if we denote market risk premium by RP_m:

$$E(r_i) = r_f + \beta_i RP_m$$

A Multifactor Approach

- in the CAPM framework, investors are rewarded only for market (non-diversifiable) risk
- if we acknowledge the existence of multiple sources of risk, the same logic should apply
- hence, investors should be rewarded for *all* types of non-diversifiable risk
- for our previous two-factor model, this implies:

$$E(r_i) = r_f + \beta_{i\,GDP} RP_{GDP} + \beta_{i\,IR} RP_{IR}$$

- we are left with finding out how to define and calculate the factor risk premiums

Arbitrage Pricing Theory (APT)

- Assumptions:
  - security returns can be described by a (multi-) factor model
  - there are sufficient securities so that firm-specific (idiosyncratic) risk can be diversified away
  - well-functioning security markets do not allow for persistent arbitrage opportunities

Arbitrage

- *arbitrage* = risk-free profits made by investors by exploiting security mispricing, *without* a net investment
- if security prices allow for arbitrage opportunities, the market is not in equilibrium
- hence, there will be pressures on prices to adjust and eliminate these risk-free profits
- *Law of One Price* = assets that are equivalent in all economically relevant aspects should have the same market price

Arbitrage Opportunities

- investors want to hold *infinite positions* in an arbitrage opportunity
- this should create pressures on prices to go up where they are too low and fall where they are too high
- in equilibrium, the market should satisfy the *no-arbitrage condition*
- note that there is a fundamental difference between risk-return dominance (CAPM) and arbitrage arguments (APT)

Well-Diversified Portfolios

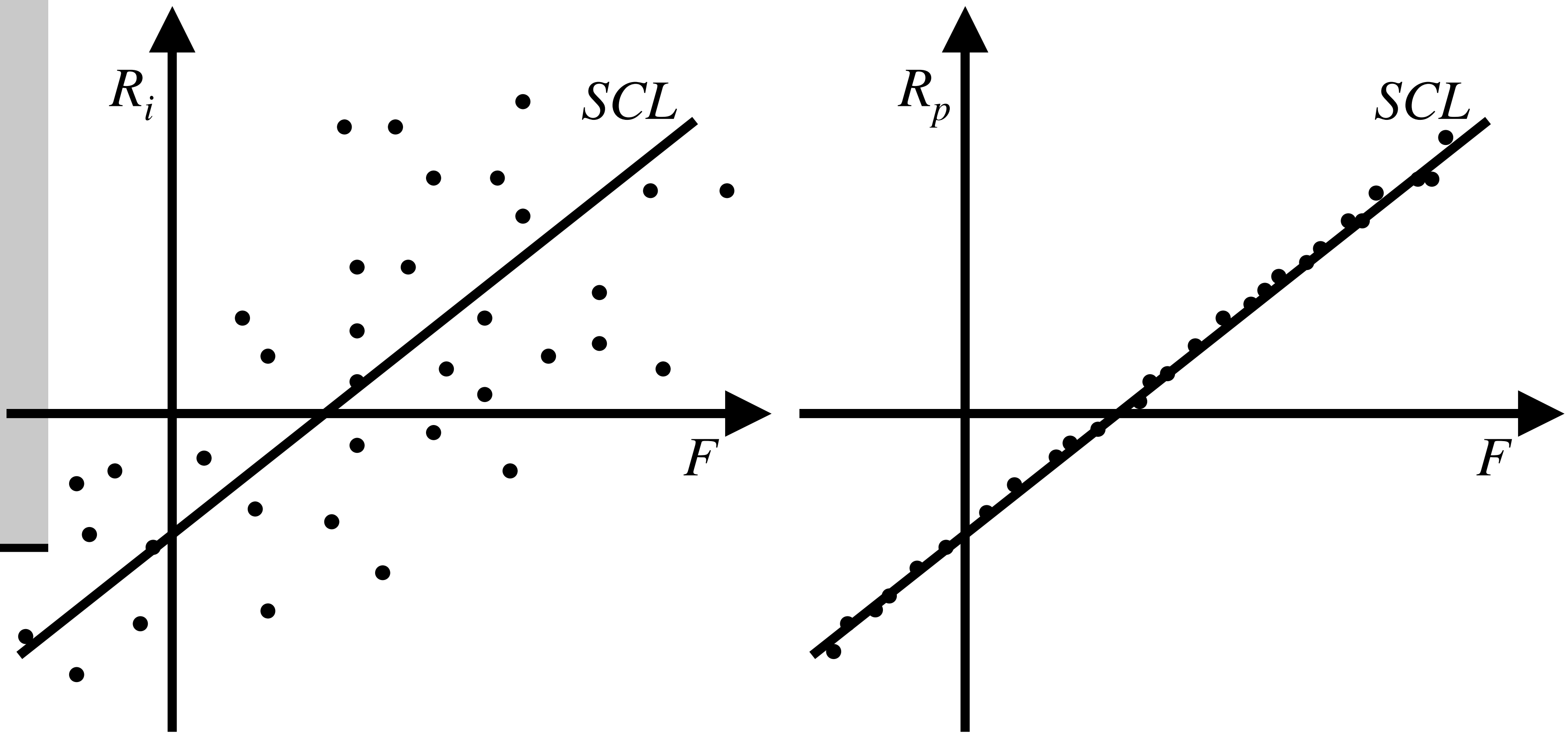
- *well-diversified portfolio* = a portfolio such that the firm-specific component of risk is negligible
- remember that, in the single-factor model,

$$\sigma_p^2 = \beta_p^2 \sigma_F^2 + \sigma_{ep}^2$$

- for a well-diversified portfolio, σ_{ep}^2 is negligible (almost zero)
- since the mean and variance of e_p are both (almost) zero, any realization of it should be almost zero
- hence, for a well-diversified portfolio,

$$r_i = E(r_i) + \beta_i F$$

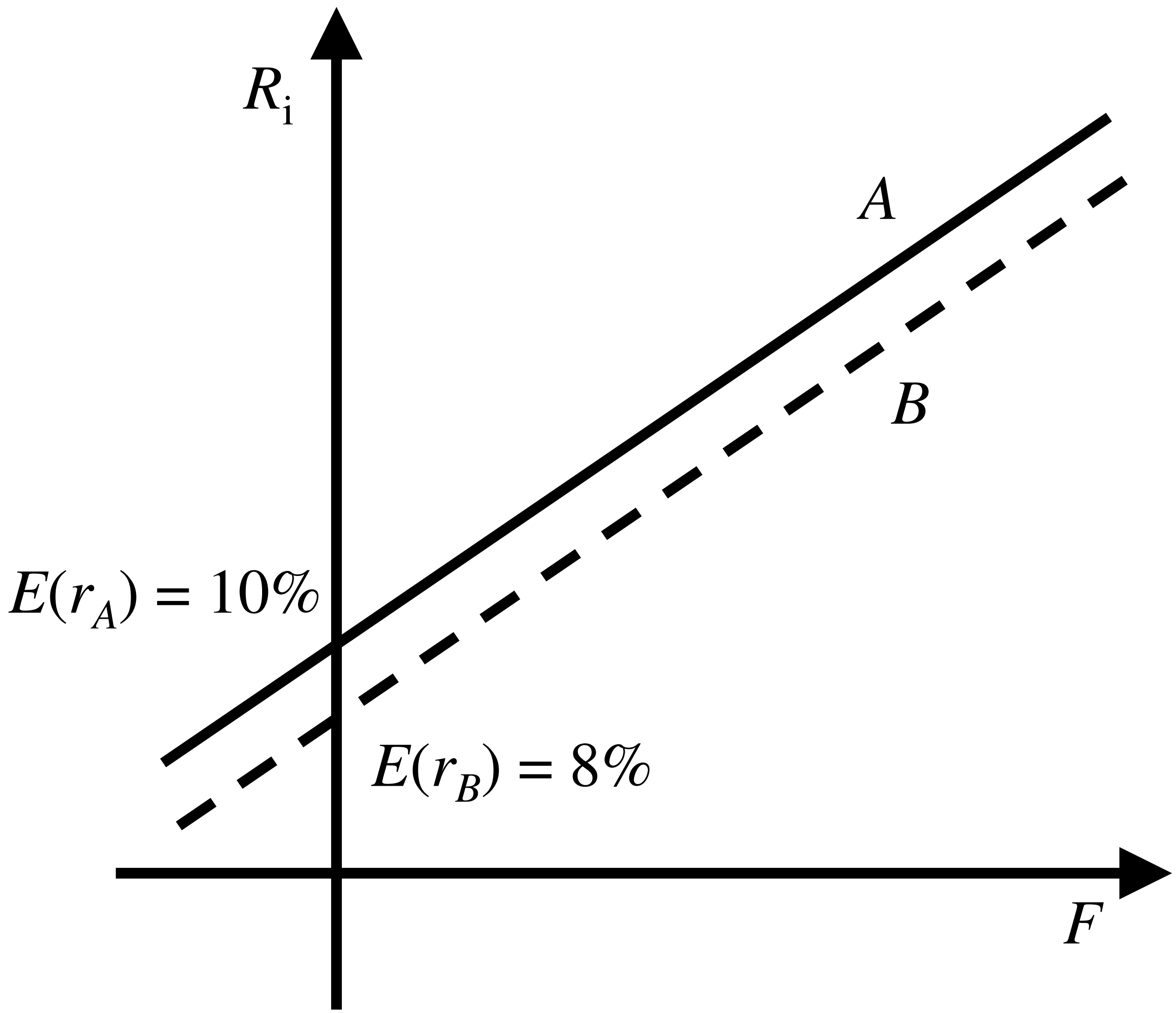
Security vs. Well-Diversified Portfolio Returns



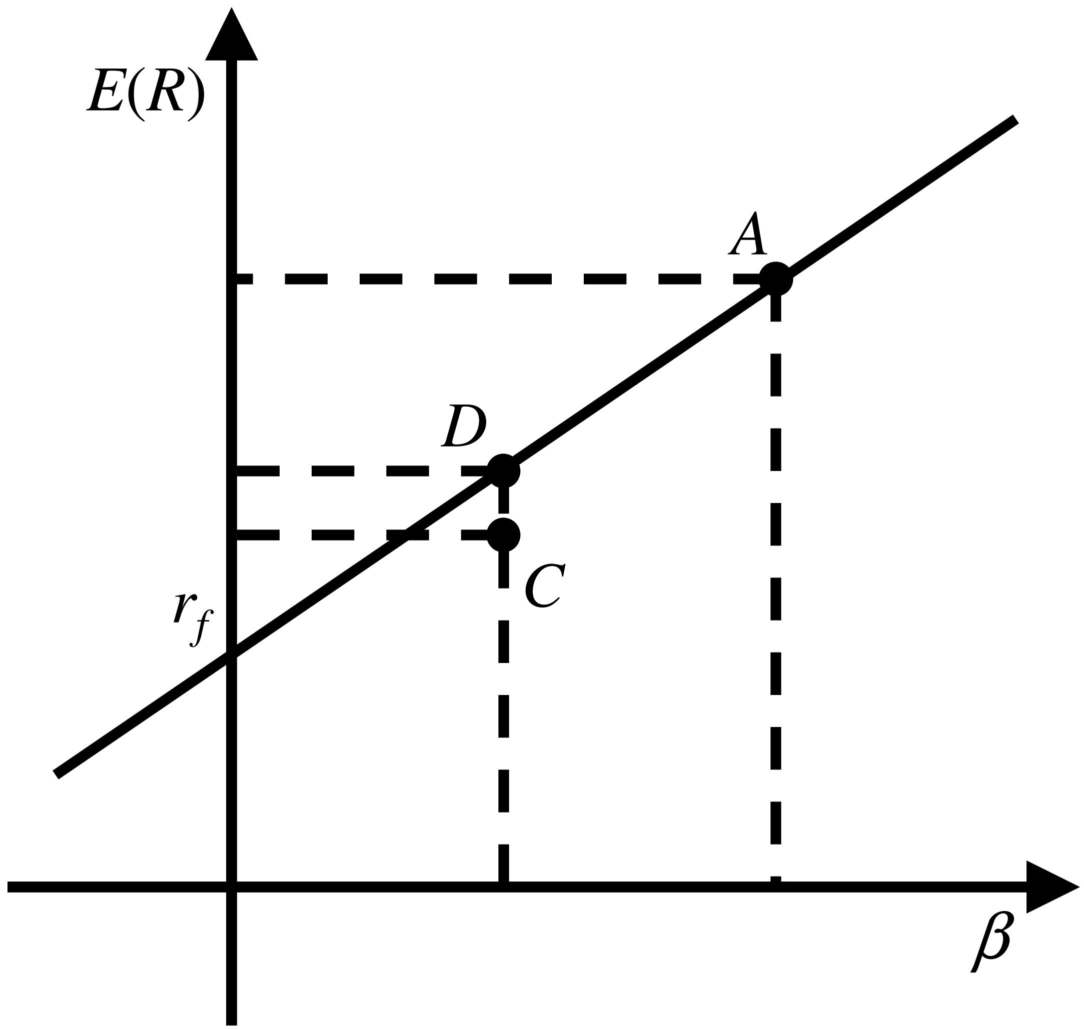
Betas and Expected Returns

- since firm-specific risk can be diversified away, investors cannot expect to be compensated for that ➔ only systematic risk should impact expected returns
- arbitrage opportunities:
  - well-diversified portfolios with same betas but *different* expected returns
  - well-diversified portfolios with risk premiums not proportional to their betas
- hence, all well-diversified portfolios should lie on the same line in the expected return-beta space

Different Expected Returns



Different Expected Return-Beta Relationship



The One-Factor Security Market Line

- suppose there is only one source of risk, that can be embedded into the market portfolio
- the above argument implies that all well-diversified portfolios lie on the same line
- the market portfolio lies on this line, so the line is defined by
  - intercept equal to r_f
  - slope equal to the risk premium on the market portfolio
- hence, a CAPM-like equation:

$$E(r_p) = r_f + \beta_p [E(r_m) - r_f]$$

APT vs. CAPM – Portfolios

- as the previous argument was based on arbitrage opportunities, it is called the *Arbitrage Pricing Theory*
- it doesn't need the strict assumptions of the CAPM
- it is not based on the market portfolio – can be any well-diversified portfolio ➔ more flexibility
- still, it yields a conclusion similar to the CAPM, at least for well-diversified portfolios

APT vs. CAPM – Individual Securities

- CAPM holds that the same relationship is true in the case of individual securities, while the APT holds it true only for well-diversified portfolios
- suppose it does not hold for *many* securities
- then it would be possible to construct a well-diversified portfolio for which the expected return-beta relationship fails
- hence, this relationship should hold for *almost all* individual securities

Factor Portfolios

- suppose again that we have more than one "market risk" factor:

$$r_i = E(r_i) + \beta_{i1} F_1 + \beta_{i2} F_2 + e_i$$

- *factor portfolio* = a well-diversified portfolio that has a beta coefficient equal to one for a specific factor and zero for all other factors
- since there are many securities, such portfolios can be constructed for each factor

Multifactor APT

- let portfolio 1 be a factor portfolio for factor 1, and portfolio 2 be a factor portfolio for factor 2
- for any well-diversified portfolio P, with factor loads β_{p1} and β_{p2}, respectively, we can construct a tracking portfolio using the two factor portfolios:
  - portfolio 1 has weight β_{p1}
  - portfolio 2 has weight β_{p2}
  - risk-free asset has weight $(1 - \beta_{p1} - \beta_{p2})$
- then, the no-arbitrage condition implies that

$$E(r_p) = r_f + \beta_{p1}\,[E(r_1) - r_f] + \beta_{p2}\,[E(r_2) - r_f]$$

Multifactor Security Market Line

- this equation is just a generalization of the CAPM equation, allowing for more than just one risk factor
- as before, it holds for all well-diversified portfolios and *almost all* individual securities
- it can be used, as in the case of CAPM, to find the “fair” return (price) on a portfolio

What Are the Factors?

- APT does not tell us *which* factors are relevant
- previous research suggests:
  - change in industrial production
  - change in expected inflation
  - change in unanticipated inflation
  - excess return of long-term corporate bonds over long-term government bonds
  - excess return of long-term government bonds over T-bills
- possible problem: identifying factors may be hindered by accidental correlations

Intertemporal CAPM (ICAPM)

- the CAPM ignores extra-market hedging needs (e.g., the need of an employee to hedge against labor income risk)
- this could cause the market portfolio not to be the risky optimal portfolio anymore
- Merton showed that these hedging demands lead to a multifactor CAPM model, where an additional risk premium is included in addition to the market portfolio:

$$E(r_i) = r_f + \beta_{im}\,[E(r_m) - r_f] + \beta_{ie}\,[E(r_e) - r_f]$$

ICAPM vs. APT

- ICAPM predicts that sources of risk against which many or dominant investors attempt to hedge will be "priced"
- such sources of risk are: labor income, prices of important consumption goods, changes in future investment opportunities
- as opposed to APT, theory "tells us" what we should look for