Arbitrage Pricing Theory and Multifactor Models of Risk and Return

Chapter 11

Single-Factor Model

- remember the single-factor model:
  \[ r_i = E(r_i) + \beta_i F + e_i \]
  where \( F \) and \( e_i \) have zero mean (as they capture surprise changes in the factors)
- this allows the decomposition of risk into market and firm-specific components
- however, this decomposition is overly simplistic:
  - ignores certain factors (e.g., industry-specific)
  - assumes all stocks respond the same to common factors embodied in \( F \)

Multifactor Models

- we should allow for different stocks to have different sensitivities to different types of market-wide shocks (e.g., inflation, business cycles, interest rates etc.)
- multifactor models = models that allow for different sensitivities to different factors
- they can provide a better description of security returns

Example – A Two-Factor Model

- suppose we believe the only macroeconomic sources of risk are business cycles (GDP) and interest rates fluctuations (IR)
- rates of return should then respond to unanticipated changes in both factors:
  \[ r_i = E(r_i) + \beta_{GDP} GDP + \beta_{IR} IR + e_i \]
- the beta coefficients are called factor sensitivities, factor loadings or factor betas
- example: compare the sensitivity of returns on an utility company and an airline company

Determining \( E(r_i) \)

- our equation is just a description of returns, there is really no theory behind it
- where does \( E(r) \) come from?
- we need a theory of market equilibrium
- CAPM is a theory of market equilibrium, but it only values aggregated market risk
  \[ E(r) = r_f + \beta [E(r_m) - r_f] \]
  or, if we denote market risk premium by \( RP_m \):
  \[ E(r) = r_f + \beta RP_m \]

A Multifactor Approach

- in the CAPM framework, investors are rewarded only for market (non-diversifiable) risk
- if we acknowledge the existence of multiple sources of risk, the same logic should apply
- hence, investors should be rewarded for all types of non-diversifiable risk
- for our previous two-factor model, this implies:
  \[ E(r) = r_f + \beta_{GDP} RP_{GDP} + \beta_{IR} RP_{IR} \]
  - we are left with finding out how to define and calculate the factor risk premiums
Arbitrage Pricing Theory (APT)

Assumptions:
- Security returns can be described by a (multi-)factor model.
- There are sufficient securities so that firm-specific (idiosyncratic) risk can be diversified away.
- Well-functioning security markets do not allow for persistent arbitrage opportunities.

Arbitrage

- Arbitrage = risk-free profits made by investors by exploiting security mispricing, without a net investment.
- If security prices allow for arbitrage opportunities, the market is not in equilibrium.
- Hence, there will be pressures on prices to adjust and eliminate these risk-free profits.
- Law of One Price = assets that are equivalent in all economically relevant aspects should have the same market price.

Arbitrage Opportunities

- Investors want to hold infinite positions in an arbitrage opportunity.
- This should create pressures on prices to go up where they are too low and fall where they are too high.
- In equilibrium, the market should satisfy the no-arbitrage condition.
- Note that there is a fundamental difference between risk-return dominance (CAPM) and arbitrage arguments (APT).

Well-Diversified Portfolios

- Well-diversified portfolio = a portfolio such that the firm-specific component of risk is negligible.
- Remember that, in the single-factor model, \( \sigma_p^2 = \beta_p^2 \sigma_F^2 + \sigma_{ep}^2 \).
- For a well-diversified portfolio, \( \sigma_{ep}^2 \) is negligible (almost zero).
- Since the mean and variance of \( e_p \) are both (almost) zero, any realization of it should be almost zero.
- Hence, for a well-diversified portfolio, \( r_p = E(r_i) + \beta_i F \).

Security vs. Well-Diversified Portfolio Returns

- Since firm-specific risk can be diversified away, investors cannot expect to be compensated for that; only systematic risk should impact expected returns.
- Arbitrage opportunities:
  - Well-diversified portfolios with the same betas but different expected returns.
  - Well-diversified portfolios with risk premiums not proportional to their betas.
- Hence, all well-diversified portfolios should lie on the same line in the expected return-beta space.

Betas and Expected Returns

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Different Expected Returns

\[ E(r_A) = 10\% \]
\[ E(r_B) = 8\% \]

Different Expected Return-Beta Relationship

The One-Factor Security Market Line
- Suppose there is only one source of risk, that can be embedded into the market portfolio
- The above argument implies that all well-diversified portfolios lie on the same line
- The market portfolio lies on this line, so the line is defined by
  - Intercept equal to \( r_f \)
  - Slope equal to the risk premium on the market portfolio
- Hence, a CAPM-like equation:
  \[ E(r_p) = r_f + \beta_p \{ E(r_m) - r_f \} \]

APT vs. CAPM – Portfolios
- As the previous argument was based on arbitrage opportunities, it is called the Arbitrage Pricing Theory
- It doesn’t need the strict assumptions of the CAPM
- It is not based on the market portfolio – can be any well-diversified portfolio
- Still, it yields a conclusion similar to the CAPM, at least for well-diversified portfolios

APT vs. CAPM – Individual Securities
- CAPM holds that the same relationship is true in the case of individual securities, while the APT holds it true only for well-diversified portfolios
- Suppose it does not hold for many securities
- Then it would be possible to construct a well-diversified portfolio for which the expected return-beta relationship fails
- Hence, this relationship should hold for almost all individual securities

Factor Portfolios
- Suppose again that we have more than one “market risk” factor:
  \[ r = E(r) + \beta_1 F_1 + \beta_2 F_2 + \epsilon \]
- Factor portfolio = a well-diversified portfolio that has a beta coefficient equal to one for a specific factor and zero for all other factors
- Since there are many securities, such portfolios can be constructed for each factor
Multifactor APT

- let portfolio 1 be a factor portfolio for factor 1, and portfolio 2 be a factor portfolio for factor 2.
- for any well-diversified portfolio $P$, with factor loads $\beta_p^1$ and $\beta_p^2$, respectively, we can construct a tracking portfolio using the two factor portfolios:
  - portfolio 1 has weight $\beta_p^1$
  - portfolio 2 has weight $\beta_p^2$
  - risk-free asset has weight $(1 - \beta_p^1 - \beta_p^2)$
- then, the no-arbitrage condition implies that
  \[
  E(r_P) = r_f + \beta_p^1 [E(r_1) - r_f] + \beta_p^2 [E(r_2) - r_f]
  \]

Multifactor Security Market Line

- this equation is just a generalization of the CAPM equation, allowing for more than just one risk factor.
- as before, it holds for all well-diversified portfolios and almost all individual securities.
- it can be used, as in the case of CAPM, to find the “fair” return (price) on a portfolio.

What Are the Factors?

- APT does not tell us which factors are relevant.
- previous research suggests:
  - change in industrial production
  - change in expected inflation
  - change in unanticipated inflation
  - excess return of long-term corporate bonds over long-term government bonds
  - excess return of long-term government bonds over T-bills
- possible problem: identifying factors may be hindered by accidental correlations.

Intertemporal CAPM (ICAPM)

- the CAPM ignores extra-market hedging needs (e.g., the need of an employee to hedge against labor income risk).
- this could cause the market portfolio not to be the risky optimal portfolio anymore.
- Merton showed that these hedging demands lead to a multifactor CAPM model, where an additional risk premium is included in addition to the market portfolio:
  \[
  E(r_i) = r_f + \beta_{im} [E(r_m) - r_f] + \beta_{ie} [E(r_e) - r_f]
  \]

ICAPM vs. APT

- ICAPM predicts that sources of risk against which many or dominant investors attempt to hedge will be “priced”.
- such sources of risk are: labor income, prices of important consumption goods, changes in future investment opportunities.
- as opposed to APT, theory “tells us” what we should look for.