

# Option Valuation

## Chapter 21

## Intrinsic and Time Value

- *intrinsic value* of in-the-money options = the payoff that could be obtained from the immediate exercise of the option
  - for a call option: stock price – exercise price
  - for a put option: exercise price – stock price
- the intrinsic value for out-the-money or at-the-money options is equal to 0
- *time value* of an option = difference between actual call price and intrinsic value
- as time approaches expiration date, time value goes to zero

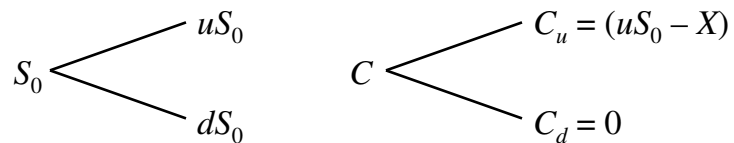
## Determinants of Option Values

	<i>Call</i>	<i>Put</i>
Stock price	+	-
Exercise price	-	+
Volatility of stock price	+	+
Time to expiration	+	+
Interest rate	+	-
Dividend rate of stock	-	+

21-3

## Binomial Option Pricing

- consider a stock that currently sells at  $S_0$
- the price can either increase by a factor  $u$  or fall by a factor  $d$  (probabilities are irrelevant)
- consider a call with exercise price  $X$  such that  $dS_0 < X < uS_0$
- hence, the evolution of the price and of the call option value is



21-4

## Binomial Option Pricing (cont.)

- now, consider the payoff from writing one call option and buying  $H$  shares of the stock, where

$$H = \frac{C_u - C_d}{uS_0 - dS_0} = \frac{uS_0 - X}{uS_0 - dS_0}$$

- the value of this investment at expiration is

	Up	Down
Payoff of stock	$HuS_0$	$HdS_0$
Payoff of calls	$-(uS_0 - X)$	0
<i>Total payoff</i>	$HdS_0$	$HdS_0$

21-5

## Binomial Option Pricing (cont.)

- hence, we obtained a risk-free investment with end value  $HdS_0$
- arbitrage argument: the current value of this investment should be equal to its present discounted value using the risk-free rate
- $H$  is called the *hedge ratio* (the ratio of the range of call option payoffs and the range of the stock price)
- the argument is based on perfect hedging, or *replication* (the payoff of the investment replicates a risk-free bond)

21-6

## Binomial Option Pricing – Algorithm

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1. given the end of period stock prices,  $uS_0$  and  $dS_0$ , calculate the payoffs of the call option,  $C_u$  and  $C_d$
2. find the hedge ratio  $H = (C_u - C_d)/(uS_0 - dS_0)$
3. calculate  $HdS_0$ , the end-of-year certain value of the portfolio including  $H$  shares of the stock and one written call
4. find the present value of  $HdS_0$ , given the riskfree interest rate  $r$
5. calculate the price of the call using the arbitrage argument:

$$HS_0 - C = PV(HdS_0)$$

21-7

## Binomial Option Pricing – Example

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- $S_0 = 100$
  - $d = .75$
  - $u = 1.5$
  - $X = 120$
  - $r = 5\%$
1.  $uS_0 = 150, dS_0 = 75$   
 $C_u = uS_0 - X = 30, C_d = 0$
  2.  $H = (C_u - C_d) / (uS_0 - dS_0) = 0.4$
  3.  $HdS_0 = 30$
  4.  $PV(HdS_0) = HdS_0 / (1 + r) = 28.57$
  5.  $HS_0 = 0.4 \cdot 100 = 40$   
 $C = HS_0 - PV(HdS_0) = 11.43$

21-8

## Generalized Binomial Option Pricing

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- the binomial model can be expanded to more than one period
- in this case, we would need to find the hedging ratio  $H$  at every node in the tree
- thus, we can construct, at each point in time, a perfectly hedged portfolio – *dynamic hedging*
- some of the nodes will be shared by different branches (e.g., the “up and down” scenario would yield the same price as the “down and up” scenario)
- although numerous and tedious calculations, can “easily” program into a computer

21-9

## Black-Scholes Valuation Model

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- Assumptions
  - European call option
  - underlying asset does not pay dividends until expiration date
  - both the (riskfree) interest rate  $r$  and the variance of the return on the stock  $\sigma^2$  are constant
  - stock prices are continuous (no sudden jumps)

21-10

## Black-Scholes Valuation Model (cont.)

### ■ Formula

- the current price of the call option is

$$C_0 = S_0 N(d_1) - X e^{-rT} N(d_2)$$

where:

- $S_0$  is the current price of the stock
- $X$  is the exercise price
- $T$  is the time until maturity of option (in years)
- $e = 2.71828$  is the base of the natural logarithm
- $N(\cdot)$  is the probability from a standard normal distribution

$$d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

21-11

## Black-Scholes Formula – Example

- $S_0 = 100$
- $X = 95$
- $r = 10\%$  per year
- $T = 0.25$  years (one quarter)
- $\sigma = 0.50$  (50% per year)
- $d_1 = \frac{\ln(100/95) + (0.10 + 0.5^2 / 2)0.25}{0.5\sqrt{0.25}} = 0.43$
- $d_2 = 0.43 - 0.5\sqrt{0.25} = 0.18$
- $N(d_1) = N(0.43) = 0.6664$ ,  $N(d_2) = N(0.18) = 0.5714$
- $C_0 = 100 \cdot 0.6664 - 95 \cdot e^{-0.10 \cdot 0.25} \cdot 0.5714 = \$13.70$

21-12

## Black-Scholes Formula – Put Options

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- to find the value of a European put option, we can use the put-call parity theorem:

$$P_0 = C_0 - S_0 + PV(X)$$

where the present value of  $X$  is calculated in continuous time:

$$PV(X) = X e^{-rT}$$

- this yields the formula:

$$P_0 = X e^{-rT} [1 - N(d_2)] - S_0 [1 - N(d_1)]$$

21-13

## Implied Volatility

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- the Black-Scholes formula is based on four observed variables ( $S_0$ ,  $X$ ,  $T$  and  $r$ ) and one unobserved variable ( $\sigma$ )
- we can estimate  $\sigma$  from historical data
- alternatively, we can calculate the value of  $\sigma$  that equates the Black-Scholes value of a call to the observed value of a call → *implied volatility*
- investors would buy the call option if they think the actual standard deviation of the stock is higher than the implied volatility

21-14

# Delta

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- the *delta* of an option is the change in the price of an option due to a \$1 increase in the stock price
- it summarizes the exposure to stock price risk
- it is the same as the hedge ratio in the binomial model
- for a call option,  $delta = N(d_1) > 0$
- for a put option,  $delta = N(d_1) - 1 < 0$