Answer Key

1. D
2. D
3. C
4. C
5. B
6. D
7. D
8. D
9. B
10. B
11. B
12. B
13. E
14. A
15. C
16. A
17. C
18. D
19. A
20. C
21. C
22. D
23. E
24. A
25. E
26. C
27. D
28. A
29. C
30. D

Question 1 (*32 points*)

Briefly explain the following:

(i) (*4 points*) the concept of diversification and how it can help reduce the risk of a portfolio. Make sure to explain which types of risk can and cannot be eliminated through diversification and why.

Answer: Diversification refers to investing in many assets in order to reduce the overall risk of the final portfolio. The purpose of diversification is to eliminate firm-specific risk (risk due to factors inherent to a particular company). However, market risk cannot be eliminated, since it affects all the assets included in the portfolio.

(ii) (*7 points*) why selling short is "more risky" than buying on margin. Also remember to describe how each of these transactions works.

Answer: In order to conduct a short sale, investors borrow stock from their brokers and sell it on the market, hoping for the price to fall. On a later date, they need to repurchase the stock on the market and return it to the brokers. Usually, brokers charge an interest for the loan and require a collateral to be kept in the investors' account (margin), the value of which should not fall below a certain level (maintenance margin).

Buying on margin works in a similar way, with investors borrowing money in order to buy stock that they hope to sell for a higher price in the future. In this case, the collateral is a fraction of the purchase price, and the brokers contribute the rest.

Short sales are profitable when the price of the stock rises, while buying on margin is profitable in the opposite case. Since the stock price can rise to infinity, short sales can potentially lead to infinite losses. In contrast, investors buying on margin can only incur finite losses. Hence, short sales are more "risky" than buying on margin.

(iii) (*4 points*) why buying options is more risky than buying stock.

Answer: The rate of return from buying options is -100% if the option is not exercised and positive if the option is exercised. In general, there is a non-trivial chance that an option will not be exercised. In contrast, the chance that the rate of return from buying stock is -100% is very slim (the stock price would need to go to zero, or the company to go bankrupt without any value left for stockholders). Therefore, since the risk of losing all the investment is (much) higher for an option than for stock, options are more risky investments than stock.

(iv) (*5 points*) the concept of arbitrage. Also give an example of an arbitrage opportunity and describe how the market would eliminate it.

Answer: An arbitrage is an risk-free strategy that yields profits without any net investment. An example of an arbitrage opportunity would be if the same asset would trade on two exchanges at different prices, at the same time. An investor could then take advantage of this by buying on the exchange where the asset sells cheaper and simultaneously selling on the exchange where the asset sells more expensive. In this case, there is no net investment (the investor uses the proceeds from the sale to buy the asset) and no risk (the transactions are conducted at the same time). However, as the investor would want to trade this asset as much as possible (since the profit depends on the volume of the transaction), he or she would drive up the demand on the "cheap market" and the supply on the "expensive market". This would cause the price to rise on the cheap market and to rise on the expensive market, until they equalize and the arbitrage opportunity disappears.

(v) (*6 points*) how the strike price influences the premium of a put and of a call option.

Answer: A put option is exercised when the strike price is higher than the market price of the stock. Therefore, the higher the strike price, the higher the chance that the option will be exercised and, hence, the more desirable the option. This means that put options with higher strike prices have higher demand, which leads to higher prices (premiums).

Conversely, a call option is exercised when the strike price is lower than the market price of the stock. In this case, the higher the strike price, the lower the chance the option will be exercised and the less desirable the option. Thus, the demand for call options with high strike prices will be lower, meaning that their price (premium) will also be lower.

(vi) (*6 points*) the semi-strong version of the efficient market hypothesis and how it invalidates (or not) technical and fundamental analysis, which you should also define.

Answer: According to the semi-strong version of the EMH, stock prices reflect all the information that can be extracted from past trading data and performance, as well as from the fundamental data (dividend prospects, quality of management, risk evaluation of the firm etc.) on the firm.

Technical analysis is a technique for forecasting stock prices based on identifying patterns using past trends and applying them to the current situation on the market. If the semi-strong version of the EMH holds, such information would already be included in prices and hence technical analysis would not be able to reveal any predictable moves in prices.

Fundamental analysis is a forecasting technique based on analyzing the fundamental data of the firm. This kind of information would still be included in prices, according to the semi-strong form of the EMH, so this analysis would also be fruitless.

Question 2 (*8 points*)

There are three assets on the market: a Treasury bill that offers an interest rate $r = 5\%$, a municipal bond with an interest rate $r_m = 8\%$ and a stock with an expected return $E(r) = 10\%$.

(i) (*3 points*) When asked to choose between the corporate bond and the stock, which one would a risk-neutral investor pick (assume the tax on interest and on stock-market earnings is the same)? Justify your answer.

Answer: A risk-neutral investor only cares about returns and not the risks involved. Therefore, such an investor would choose the stock, since its expected return is higher than the return on the corporate bond.

(ii) (*2 points*) In what tax bracket should be an investor who is indifferent between municipal bond and the corporate bond?

Answer: The tax bracket is given by

$$t = 1 - \frac{r_m}{r_b} = 1 - \frac{5}{8} = 1 - 0.625 = 0.375 \Rightarrow \boxed{t = 37.5\%.}$$

(iii) (*3 points*) Calculate the equivalent taxable yield for an investor facing a 20% tax rate. Which of the two bonds would he or she choose?

Answer: The equivalent taxable yield is

$$r = \frac{r_m}{1-t} = \frac{5}{1-0.20} = \frac{5}{0.8} \Rightarrow \boxed{r = 6.25\%.}$$

Since the equivalent taxable yield is lower than the yield on the corporate bond, the investor would prefer the corporate bond, even though its interest is taxable.

Question 3 (*17 points*)

Stocks A and B have expected returns equal to $E(r_A) = 6.6\%$ and $E(r_B) = 8.6\%$ respectively. Their covariances with the return on the market portfolio are $Cov(r_A, r_M) = 90$ and $Cov(r_B, r_M) = 140$. (Note: be careful if you want to work with decimals rather than percentages.) The variance of the return on the market portfolio is $\sigma_M^2 = 100$.

(i) (*2 points*) What are the beta coefficients for stocks A and B?

Answer: The beta coefficients for the two stocks are

$$\boxed{\beta_A = \frac{Cov(r_A, r_m)}{\sigma_m^2} = \frac{90}{100} = 0.9,}$$
$$\boxed{\beta_B = \frac{Cov(r_B, r_m)}{\sigma_m^2} = \frac{140}{100} = 1.4.}$$

(ii) (*8 points*) Using the CAPM model for the two stocks, calculate the risk-free rate and the expected return on the market portfolio.

Answer: The CAPM equation is:

$$E(r) = r_f + \beta \left[E(r_m) - r_f\right].$$

Applying that to the two stocks gives a system of two equations in two unknowns ($E(r_m)$

and r_f):

$$\begin{cases} E(r_A) = r_f + \beta_A \left[E(r_m) - r_f\right] \\ E(r_B) = r_f + \beta_B \left[E(r_m) - r_f\right] \end{cases} \Rightarrow \begin{cases} 6.6 = r_f + 0.9 \left[E(r_m) - r_f\right] \\ 8.6 = r_f + 1.4 \left[E(r_m) - r_f\right] \end{cases}$$

$$\Rightarrow \begin{cases} 6.6 = r_f + 0.9E(r_m) - 0.9r_f \\ 8.6 = r_f + 1.4E(r_m) - 1.4r_f \end{cases} \Rightarrow \begin{cases} 6.6 = 0.9E(r_m) + 0.1r_f \\ 8.6 = 1.4E(r_m) - 0.4r_f \end{cases}$$

The first equation can be solved for $E(r_m)$:

$$6.6 - 0.1r_f = 0.9E(r_m) \Rightarrow E(r_m) = \frac{6.6 - 0.1r_f}{0.9},$$

which can now be replaced in the second equation to solve for r_f:

$$8.6 = 1.4 \cdot \frac{6.6 - 0.1r_f}{0.9} - 0.4r_f \Rightarrow 8.6 = \frac{1.4 \cdot (6.6 - 0.1r_f) - 0.9 \cdot 0.4r_f}{0.9}$$
$$\Rightarrow 8.6 \cdot 0.9 = 1.4 \cdot 6.6 - 1.4 \cdot 0.1r_f - 0.36r_f \Rightarrow 7.74 = 9.24 - 0.14r_f - 0.36r_f$$
$$\Rightarrow 0.50r_f = 9.24 - 7.74 \Rightarrow 0.50r_f = 1.5 \Rightarrow r_f = \frac{1.5}{0.50} \Rightarrow \boxed{r_f = 3\%}$$

This value can now be used in the equation for $E(r_m)$:

$$E(r_m) = \frac{6.6 - 0.1r_f}{0.9} = \frac{6.6 - 0.1 \cdot 3}{0.9} = \frac{6.6 - 0.3}{0.9} = \frac{6.3}{0.9} \Rightarrow \boxed{E(r_m) = 7\%.}$$

(iii) (*2 points*) Portfolio P is constructed by combining stocks A and B with weights $w_A = 0.8$ and $w_B = 0.2$. What are the expected return and the beta of the portfolio?

Answer: The beta and expected return of the portfolio are given by

$$\beta_p = w_A\beta_A + w_B\beta_B = 0.8 \cdot 0.9 + 0.2 \cdot 1.4 = 0.72 + 0.28 \Rightarrow \boxed{\beta_p = 1,}$$
$$E(r_p) = w_A E(r_A) + w_B E(r_B) = 0.8 \cdot 6.6 + 0.2 \cdot 8.6 = 5.28 + 1.72 \Rightarrow \boxed{E(r_p) = 7\%.}$$

(Note: you could have mentioned that the beta for the portfolio is the same as the beta for the market portfolio, therefore their expected returns must be the same. You could have made a similar argument for the equality of the beta coefficients if you calculated the expected return first.)

(iv) (*5 points*) Suppose you have some information that makes you believe the return on stock A should actually be $E^a(r_A) = 8\%$ and the return on stock B should be $E^a(r_B) = 8\%$. What are the alpha coefficients for the two stocks? Are they overpriced, underpriced or fairly priced? How would you take advantage of any mispricing you found?

Answer: The alpha coefficients for the two stocks are:

$$\alpha_A = E^a(r_A) - E(r_A) = 8 - 6.6 \Rightarrow \boxed{\alpha_A = 1.4\%,}$$
$$\alpha_B = E^a(r_B) - E(r_B) = 8 - 8.6 \Rightarrow \boxed{\alpha_B = -0.6\%.}$$

Therefore, stock A is underpriced and stock B is overpriced. You would want to buy stock A and sell stock B short, so that you gain once the market re-aligns their prices (i.e., stock A's price rises and stock B's falls).

Question 4 (*13 points*)

Stock A currently sells for $30 a share and the ongoing (risk-free) interest rate is 4%.

(i) (*6 points*) Suppose you wanted to construct a covered call by buying stock and selling a call option with maturity in six months, strike price $X = \$32$, and premium $C = \$3$. Derive and graph the payoff and the profit from this investment strategy. When would you want to pursue such an investment?

Answer: The payoff and the profit from the covered call are shown in the table below (they are displayed in figure 1(a)):

	$S_T \leq X = \$32$	$S_T > \$32$
Payoff of stock holding	S_T	S_T
Payoff of call writing	0	$\$32 - S_T$
Payoff of covered call	S_T	$32
Profit of covered call	$S_T + \$3$	$\$32 + C = \$35$

You would want to use this strategy when you plan to sell the stock at the strike price anyway, but you would want to boost your portfolio with the premium from the call options.

(ii) (*7 points*) Now suppose you wanted to construct a protective put by buying stock and buying a put option with the same characteristics as the call option in the previous part. What would be the premium on this put option? Derive and graph the payoff and the profit from this investment strategy. When would you want to pursue such an investment?

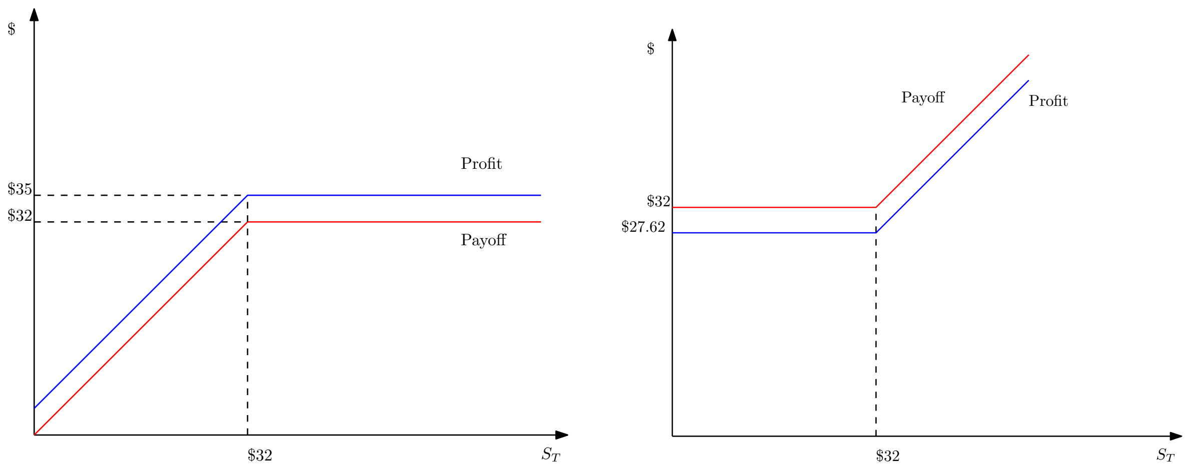
Answer: The premium on the put option can be found using the put-call parity:

$$P = C + \frac{X}{(1+r_f)^T} - S_0 = 3 + \frac{32}{(1+0.04)^{1/2}} - 30 \Rightarrow \boxed{P = \$4.38.}$$

Then, the payoff and the profit from the protective put are shown in the table below (they are plotted in figure 1(b)):

	$S_T \leq X = \$32$	$S_T > \$32$
Payoff of stock holding	S_T	S_T
Payoff of put holding	$\$32 - S_T$	0
Payoff of protective put	$32	S_T
Profit of protective put	$\$32 - P = \$27.62$	$S_T - \$4.38$

You would want to use such a strategy when you want to get protection against price drops. With this strategy, the value of your stock would never go below $27.62 per share.



(a) Covered call

(b) Protective put

Figure 1: Payoffs and profits (per share) of the two investment options.

Extra Credit Question *(extra 15 points)*

Consider the CAPM model.

(i) (*3 points*) Define the separation property and describe the "optimal" risky portfolio that all investors would choose.

Answer: The separation property refers to the fact that an individual's investment decision can be split into two parts: an "objective" part and a "subjective" part. The objective decision is the choice of a risky portfolio to invest in, and all the investors would choose the same portfolio (the point of tangency from r_f to the Minimum-Variance Frontier). The subjective decision involves the choice of a particular complete portfolio (a point on the Capital Market Line).

Since all investors choose the same optimal risky portfolio, this portfolio is the market portfolio, i.e., a portfolio that includes all the assets traded on the market, in the same proportion as their market value's proportion in the total market value.

(ii) (*2 points*) What is the Security Market Line? Which assets does it plot? Write down its equation.

Answer: The Security Market Line describes the relationship between expected return and risk according to the CAPM model. Since this relationship applies to all the assets on the market, the line plots all the assets that are available or can be constructed on the market. Its equation is:

$$E(r_i) = r_f + \beta_i[E(r_m) - r_f].$$

(iii) (*2 points*) Explain how one would use the CAPM model to find underpriced securities. (Do not forget to show how to calculate the *alpha* of a security.)

Answer: The CAPM model give the fair price of an asset. If we have a personal estimation of the return of a security, $E^a(r_i)$, then we can compare this estimation to the return predicted by the CAPM model to calculate the *alpha* of the security:

$$\alpha_i = E^a(r_i) - E(r_i).$$

If α_i is positive, then the security is underpriced and, thus, desirable to buy. If α_i is negative, then the security is overpriced.

(iv) (*3 points*) Explain the differences between the Security Market Line and the Capital Market Line and why they use different measures of risk.

Answer: The Security Market Line describes the relationship between expected return and risk (as measured by beta) for all the assets on the market. Since investors would not want to invest in assets on their own, but as part of a larger portfolio, they are not interested in the *total* risk of an individual asset, but rather in their contribution to the final investment. This contribution is captured by beta, and thus this is the correct measure of risk in this setup.

The Capital Market Line describes the relationship between expected return and *total* risk (as measured by standard deviation) for complete portfolios. Since these are the final investment of an individual, total risk (variance or standard deviation) is the relevant measure of risk.