Econ435 – Financial Markets and the Macroeconomy
Solutions to Problem Set 3

Question 1 (*4 + 3 points*)

(i) (*2 points*) The certainty equivalent is the risk-free rate of return that gives the investor the same utility as investing in the stock A. The utility to investor X from investing in the stock A is:

$$U(A) = E(r_A) - 0.005A\sigma_A^2 = 10 - 0.005 \cdot 5 \cdot 17^2 = 10 - 7.225 = 2.78,$$

and the utility from the certainty equivalent is $U(ce) = ce$, since the certainty equivalent is riskless. Hence,

$$U(ce) = U(A) \quad \Rightarrow \quad ce = 2.78\%.$$

Investor X's certainty equivalent is lower than investor Y's (4.22%), so she is more risk averse: she is willing to give up more return in order to avoid the risk in stock A.

(ii) (*2 points*) Using the same argument as before, the utility from the certainty equivalent for investor Y is the same as her utility from investing in stock A:

$$U(ce) = U(A) \quad \Rightarrow \quad ce = E(r_A) - 0.005A\sigma_A^2 \quad \Rightarrow \quad 4.22 = 10 - 0.005 \cdot A \cdot 17^2.$$

Solving for A gives:

$$A = \frac{10 - 4.22}{0.005 \cdot 17^2} = \frac{5.78}{1.445} = 4.$$

Investor Y has a lower coefficient of risk aversion as investor X, so she is less risk averse. This confirms our finding in the previous part.

(iii) (*extra 3 points*) Using the same definition as above, we can calculate the certainty equivalent for the two investors:

$$ce_A = E(r_B) - 0.005 \cdot 5 \cdot \sigma_B^2 = 8 - 0.005 \cdot 5 \cdot 12^2 = 8 - 3.6 = 4.40\%,$$
$$ce_B = E(r_B) - 0.005 \cdot 4 \cdot \sigma_B^2 = 8 - 0.005 \cdot 4 \cdot 12^2 = 8 - 2.28 = 5.12\%.$$

The certainty equivalent of investor X is below risk-free rate (5%), so she would choose to invest in (i.e., buy) the risk-free asset rather than stock B. Investor B, however, has a certainty equivalent higher than the risk-free rate, meaning that she would choose to invest in stock B.

Question 2 (*11 + 1 points*)

(i) (*2 points*) By definition, the beta coefficient of an asset i is equal to $\beta_i = Cov(r_i, r_m)/\sigma_m^2$. Hence, the beta coefficients of two stocks A is:

$$\beta_A = \frac{Cov(r_A, r_m)}{\sigma_m^2} = \frac{243}{15^2} = 1.08.$$

The expected return of stock A is given by the CAPM model:

$$E(r_A) = r_f + \beta_A \left[E(r_m) - r_f\right] = 5 + 1.08\,(12 - 5) = 12.56\%.$$

(ii) (*2 points*) According to the CAPM model, the expected return of stock B is

$$E(r_B) = r_f + \beta_B \left[E(r_m) - r_f\right] \quad \Rightarrow \quad 10.6 = 5 + \beta_B\,(12 - 5)\,.$$

We can solve for β_B to get

$$\beta_B = \frac{10.6 - 5}{12 - 5} = \frac{5.6}{7} = 0.8.$$

Since the beta coefficient of stock B is lower than the one of stock A, stock B is less risky.

(iii) (*2 points*) The beta coefficient of a portfolio is just the weighted sum of the beta coefficients of the assets included in the portfolio. Hence, for the first portfolio (say P),

$$\beta_P = w_A\beta_A + w_B\beta_B = 0.4 \cdot 1.08 + 0.6 \cdot 0.8 = 0.912.$$

Then the "fair" return on portfolio P is

$$E(r_P) = r_f + \beta_P \left[E(r_m) - r_f\right] = 5 + 0.912\,(12 - 5) = 11.384\%.$$

(iv) (*2 points*) Similarly, for the second portfolio (say Q),

$$\beta_Q = w_A\beta_A + w_B\beta_B = 0.8 \cdot 1.08 + 0.2 \cdot 0.8 = 1.024.$$

Then the "fair" return on portfolio Q is

$$E(r_Q) = r_f + \beta_Q \left[E(r_m) - r_f\right] = 5 + 1.024\,(12 - 5) = 12.168\%.$$

(v) (*1 point*) The security market line is shown in figure 1, together with the five assets in this example: stocks A and B, portfolios P and Q, and the market portfolio M.

(vi) (*2 points*) By definition, the alpha coefficient of any asset i is the difference between the actual and the "fair" expected return: $\alpha_i = E^a(r_i) - E(r_i)$. Hence, the alpha coefficients of stocks A and B are

$$\alpha_A = E^a(r_A) - E(r_A) = 15\% - 12.56\% = 2.44\%,$$
$$\alpha_B = E^a(r_B) - E(r_B) = 11\% - 10.60\% = 0.40\%.$$

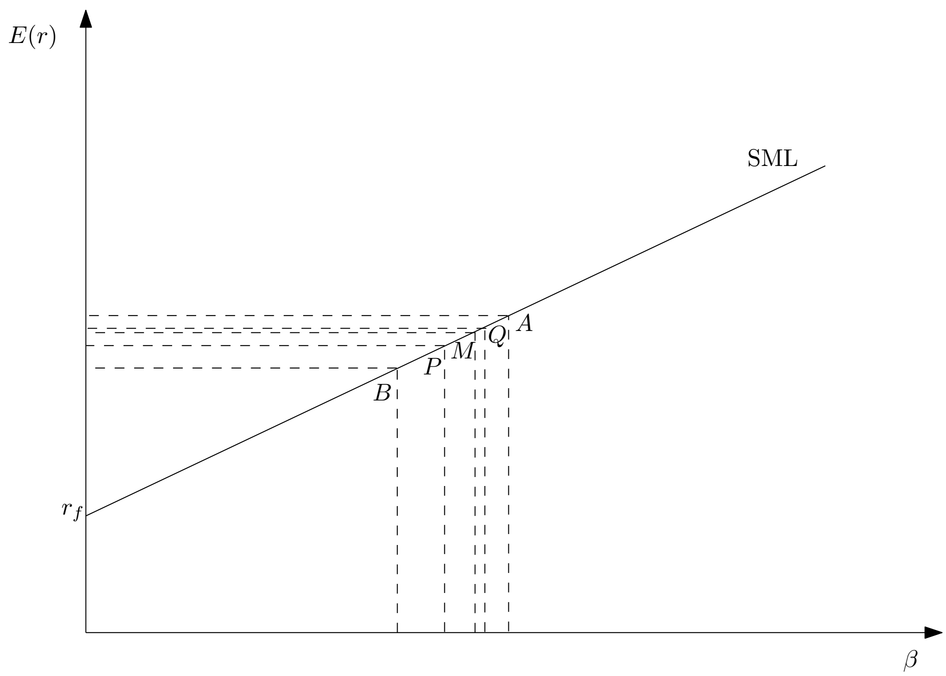

Figure 1: Security Market Line

Both stocks have negative alphas, meaning that they are both underpriced and thus desirable investments. Preference of one over the other depends on the degree of risk aversion of individual investors.

(vii) (*extra 1 point*) According to the CAPM model, the only factor that influences the (expected) return on an asset is its risk as measured by beta. Therefore, no other factor (like a stock-split) should influence *returns*. Stock prices will change to reflect the stock split, but *returns* will not be affected. In conclusion, the expected return of stock A should not change.

Question 3 (*2 + 1 points*)

(i) (*1 point*) In the case when there is no risk-free asset, the CAPM still holds with the zero-beta asset replacing the risk-free asset. So, the expected return of stock B is given by

$$E(r_B) = E(r_{zm}) + \beta_B \left[E(r_m) - E(r_{zm})\right] = 4 + 1.5\,(10 - 4) = 4 + 9 = 13\%.$$

(ii) (*1 point*) We can use the same extension of the CAPM model to relate the expected return of stock A to its risk (beta):

$$E(r_A) = E(r_{zm}) + \beta_A \left[E(r_m) - E(r_{zm})\right] \quad \Rightarrow \quad 8 = 4 + \beta_A\,(10 - 4)\,.$$

Solving for β_A yields:

$$\beta_A = \frac{8-4}{10-4} = \frac{4}{6} = 0.67.$$

(iii) (*extra 1 point*) We can use yet another extension of the CAPM model, taking into account the liquidity premium, in which case the expected return of stock B is

$$E(r_B) = E(r_{zm}) + \beta_B\,[E(r_m) - E(r_{zm})] + f(c_B) = 4 + 1.5\,(10-4) + 2 = 13 + 2 = 15\%.$$

Question 4 (*3 points*)

The only type of risk that is left in well diversified portfolios is market risk. This is the type of risk for which the market rewards investors. Therefore, the total risk of the portfolio, as measured by its variance or standard deviation, is a good measure of risk.

Not-so-well diversified portfolios or individual assets, however, include both market risk and at least some firm-specific risk. While the market rewards investors for the former, it does not for the latter, since they could always eliminate it by diversification. Therefore, it is assumed that investors would consider these assets only as part of a larger investment, so that they can eliminate the firm-specific risk in the overall asset holdings. In this case, "risk" should be interpreted only as the contribution of the asset to the risk of the final portfolio, which is exactly what beta measures (since each investor will hold the market portfolio). In conclusion, beta is the right measure of risk because these assets still have individual risk, for which the market does not pay, and they should be considered as part of a larger investment.