

Econ435 – Financial Markets and the Macroeconomy

Solutions to Problem Set 4

Question 1 (16 points)

(i) (1 point) The current price of the stock is $200,000/2,500 = \$80$ per share.

(ii) (4 points) The payoff and the profit of the protective put, as functions of the future price S_T , are derived in the table below:

	$S_T \leq X_p = \$100$	$S_T > \$100$
Payoff of stock holding	S_T	S_T
Payoff of put holding	$100 - S_T$	0
Payoff of protective put	\$100	S_T
Profit of protective put	$100 - P = \$90$	$S_T - \$10 > \90
Total value of investment	$2,500 \cdot \$90 = \$225,000$	$2,500(S_T - \$10) > \$225,000$

Recall that the profit and payoff of the investment strategy are quoted on a per-share basis. Since the Smiths hold 2,500 shares, the total value of the investment strategy will be 2,500 times the profit of one protective put. Graphically, the payoff and profit of the protective put are represented in figure 1(a).

Notice that the Smiths might not be able to afford their dream house. In fact, for them to end up with the sum required to buy the house, they would need that $2,500(S_T - 10) = 240,000$. This means that they need the stock price to be at least $(240,000 + 2,500 \cdot 10)/2,500 = \106 per share.

(iii) (4 points) The payoff and the profit of the covered call, as functions of the future price S_T , are derived in the table below:

	$S_T \leq X_c = \$95$	$S_T > \$95$
Payoff of stock holding	S_T	S_T
Payoff of call writing	0	$95 - S_T$
Payoff of covered call	S_T	\$95
Profit of covered call	$S_T + \$8 < \103	$95 + C = \$103$
Total value of investment	$2,500(S_T + \$8) < \$257,500$	$2,500 \cdot \$100 = \$257,500$

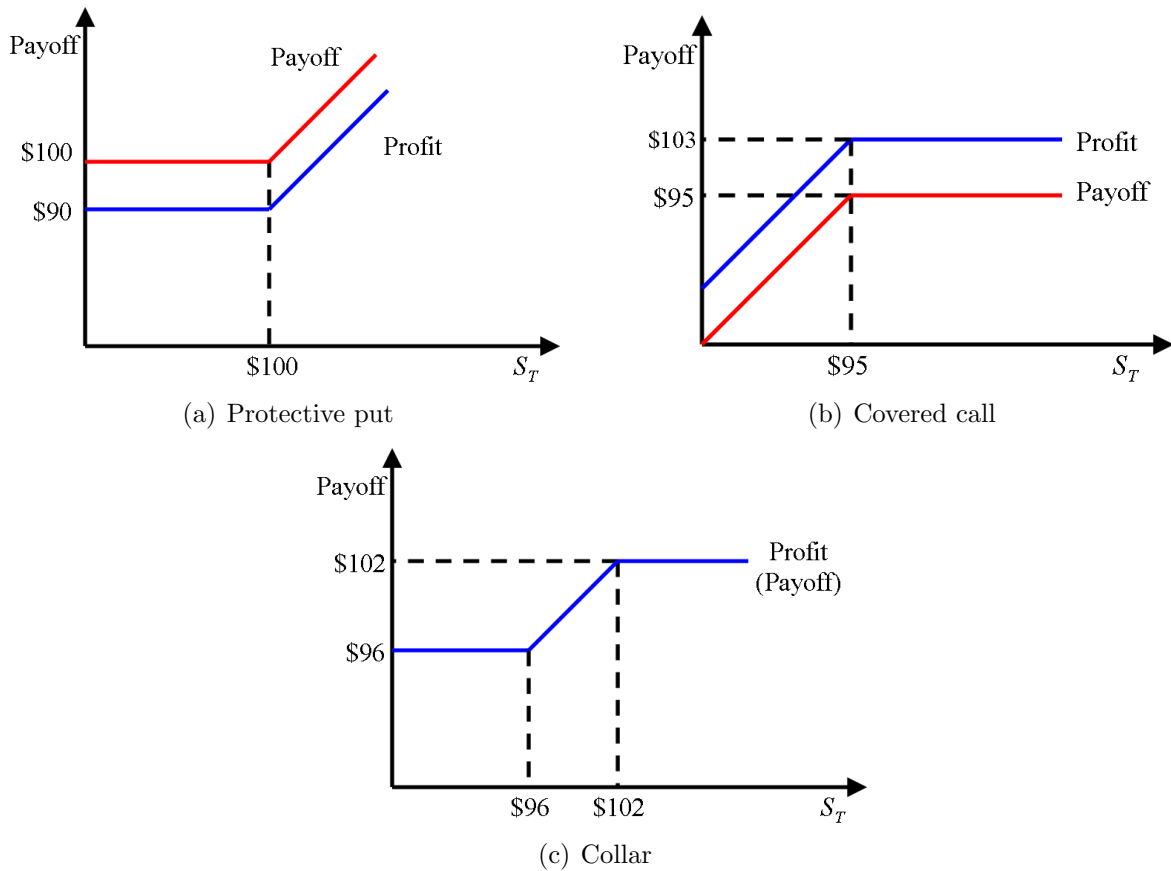


Figure 1: Payoffs and profits (per share) of the three investment options.

The graph of the payoff and profit of the covered call are represented in figure 1(b). Notice that, unlike the graph for a covered call in the lecture notes, the profit curve is above the payoff curve.

Also, it is again the case that the Smiths might not be able to afford their dream house. If they want to end up with the sum required to buy the house, they would need that $2,500(S_T+8) = 240,000$, i.e. that the stock price be at least $(240,000-2,500 \cdot 8)/2,500 = \88 per share (which is less than what they needed in the case of the protective put).

(iv) ($\frac{1}{4}$ points) The payoff and the profit of collar, as functions of the future price S_T , are derived in the table below:

	$S_T \leq X_1(\$96)$	$\$96 < S_T \leq X_2(\$102)$	$S_T > \$102$
Payoff of stock holding	S_T	S_T	S_T
Payoff of put holding	$\$96 - S_T$	0	0
Payoff of call writing	0	0	$\$102 - S_T$
Payoff of collar	$\$96$	S_T	$\$102$
Profit of collar	$\$96$	S_T	$\$102$
Total value of investment	$2,500 \cdot \$96 = \$240,000$	$2,500 \cdot S_T$	$2,500 \cdot \$102 = \$255,000$

The graph of the payoff (or profit) of the collar is displayed in figure 1(c). This time, whatever the price of the stock, the Smiths will afford their dream house, as the total end-value of the investment is at least \$240,000.

(v) (3 points) If you think the price would fall, then you should not recommend the covered call, as the Smiths would incur losses rather than preserve the value of their portfolio. The protective put is not very desirable either, since they would not be able to afford the house if the price falls (the value of the investment becomes \$225,000). Their best option would be the collar, that ensures that their portfolio would be worth \$240,000 if the stock price falls.

If you are sure that the stock price would increase, the best strategy is a protective put: its profit (hence the value of the investment) rises with the stock price. For example, if the stock price were to increase to \$120 per share, the total value of the investment would be $2,500(120 - 10) = \$275,000$. The other two investment alternatives do not increase their value as the stock price increases, as it can be seen from the graphs in figure 1.

Finally, if you are unsure about the evolution of the stock price, the best option is the collar, which protects the Smiths against price falls while ensuring a minimum value for their holdings. To achieve that, they give up the potential of higher profits when the stock price increases (the profit of the collar when the stock price is above \$95 is the lowest among the three alternatives).

Question 2 (4 + 3 points)

(i) (4 points) First, we need to calculate d_1 and d_2 :

$$d_1 = \frac{\ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} = \frac{\ln(80/75) + (0.10 + 0.25^2/2)0.5}{0.25\sqrt{0.5}} = 0.74,$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.74 - 0.25\sqrt{0.5} = 0.56.$$

The probabilities from the standard normal distribution are:

$$\begin{aligned}N(d_1) &= N(0.74) = 0.77, \\N(d_2) &= N(0.56) = 0.71.\end{aligned}$$

Now we can plug in the Black-Scholes formula and calculate the value of the call option:

$$C_0 = S_0N(d_1) - Xe^{-rT}N(d_2) = 80 \cdot 0.77 - 75 \cdot e^{-0.10 \cdot 0.5} \cdot 0.71 = \$10.74.$$

(ii) (*extra 2 points*) We can calculate the value of the put option using the put-call parity theorem:

$$P_0 = C_0 + Xe^{-rT} - S_0 = 10.74 + 75 \cdot e^{-0.10 \cdot 0.5} - 80 = \$2.08,$$

or we can apply the Black-Scholes formula for put options:

$$P_0 = Xe^{-rT}[1 - N(d_2)] - S_0[1 - N(d_1)] = 75 \cdot e^{-0.10 \cdot 0.5}(1 - 0.71) - 80(1 - 0.77) = \$2.08.$$

Question 3 (*extra 3 points*)

First, we need to calculate the possible values of the stock price: $uS_0 = 1.3 \cdot 50 = \$65$ and $dS_0 = 0.5 \cdot 50 = \$25$. Now, it is easy to see that the call option will be exercised only if the price increases, meaning that the payoffs of the call are $C_u = uS_0 - X = 65 - 55 = \10 and $C_d = \$0$. We can now calculate the hedge ratio:

$$H = \frac{C_u - C_d}{uS_0 - dS_0} = \frac{10 - 0}{65 - 25} = 0.25.$$

Hence, the risk-free portfolio would have a value of $HdS_0 = 0.25 \cdot 25 = \6.25 . The present discounted value of the value of the portfolio is

$$PV(HdS_0) = \frac{HdS_0}{1 + r} = \frac{6.25}{1 + 0.10} = \$5.68.$$

Finally, this gives us the value of the call option as

$$C = HS_0 - PV(HdS_0) = 0.25 \cdot 50 - 5.68 = \$6.82.$$